Minor Candidates as Kingmakers*

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Abstract

We consider a sequential entry model with three candidates who cannot commit to any policy announcement during the campaign. The study focuses on how a minor candidate, who wins only when unopposed, influences the electoral outcome. We show that unless the Condorcet winner (i.e., the winner in every pairwise vote) coincides with the grand winner (i.e., the winner of the three-candidate competition), the minor candidate is a kingmaker in the sense that his preferred rival wins regardless of the order of the entry decisions. To influence the outcome, the minor candidate could either (i) enter strategically without any chance to win, or (ii) enter if and only if the Condorcet winner already has entered.

Keyword: Minor Candidates, Kingmakers, Sequential Entry Decisions, Condorcet Winner, Strategic Candidacy, Threatening

JEL classification: D72, D78

1 Introduction

In political competition with more than two candidates, votes tend to be split among the candidates who display similar ideological positions. This phenomenon, known as the *spoiler effect*, has the potential to change the winner, and, thus, often leads to debates about problems with the current voting system, the intentions of the candidates, and

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so on. One of the prominent examples of the spoiler effect is the 2000 US presidential election. George W. Bush received fewer popular votes than Al Gore.[1] However, Bush eventually won the election because he prevailed in Florida, where the votes for Gore were non-negligibly split by the Independent candidate, Ralph Nader (Burden 2003). A more recent example is Maine's 2014 gubernatorial election. Although the incumbent, Paul LePage, trailed Mike Michaud according to a preelection poll, LePage was reelected because of the presence of the third candidate, Eliot Cutler, who split the leftwing vote (Nir 2013). In both elections, the third-party candidates were minor in that they had little chance of winning, but they had significant impacts on the voting outcomes.

Another factor influencing political competition is the lag between candidacy announcements. By observing competitors' entry decisions, potential candidates may plan their own entries strategically. For instance, in the 2016 US presidential election, Michael R. Bloomberg had considered running as an independent candidate given that the candidates representing the Republican and Democratic parties would repel many voters. However, Bloomberg finally decided against entering to avoid the risk of the Republican candidate winning by splitting votes for the Democratic Party in what would have become a three-candidate race (Bloomberg 2016). Likewise, a lag in candidacy causes potential candidates to care about the followers' entry decisions. Such behavior has been observed in an election for non-permanent seats in the United Nations Security Council (UNSC), which replaces half of its current members every year. A recent interesting example is Chile's decision on its UNSC candidacy. In 2006, Chile declared not to stand for the 2012 election even though no other countries had declared their candidacies for seats in 2012.[2] One of the reasons was that Chile expected that Argentina would announce its candidacy later and wanted to avoid competition with Argentina (Iwanami 2016). A question arises from these observations: under what condition does a minor candidate have a strong

[1] Bush and Gore obtained 47.87% and 48.38% of the total votes, respectively.
[2] Instead, Chile declared its candidacy for the 2013 election.

influence on leaders or followers?

As the first step to investigate these phenomena, we consider a sequential entry model, where (i) the decision order is fixed exogenously, (ii) there are three potential candidates, and (iii) one of the candidates is called *minor* because he never wins unless no competitor enters the race. Our main result insists that as long as the minor candidate splits the others' votes non-negligibly, he can influence the electoral outcome as a "kingmaker" in the following sense. The rival of the minor candidate might be a *Condorcet winner* (hereafter, CW), who is the winner in every pairwise vote, or a *grand winner* (hereafter, GW), who is the winner in a three-candidate race. Unless CW coincides with GW, the candidate whom the minor candidate prefers to win is the election's winner regardless of the order of decision making or the preferences of the other candidates. The order of decision making is relevant for *how* the minor candidate influences the electoral outcome. In particular, when the minor prefers GW to win and decides before CW, he strategically enters without any chance to win, which can crowd out CW from the set of entered candidates. On the other hand, if the minor prefers GW to win and decides after CW, he "threatens" CW in the sense that the minor enters after CW's entry, which induces a three-candidate race.[3]

Our paper is related to two research streams. First, our model is based on the seminal works of Osborne & Slivinski (1996) and Besley & Coate (1997), who develop the citizen–candidate model, wherein citizens choose endogenously whether to stand for election and entry is costly.[4] The citizen–candidate model usually assumes that no policy announcement during the campaign is credible, and the winning candidate then implements his ideal policy. The assumption of no commitment simplifies the analysis by focusing on entry decisions without the complications of a policy announcement. Second, Palfrey

[3]If the minor prefers CW to win, then he simply exits to assure the winning of CW.

[4]Osborne & Slivinski (1996) restrict attention to one-dimensional spatial competitions with a continuum of citizens who vote sincerely. As an alternative, Besley & Coate (1997) consider environments wherein finite sets of citizens vote strategically across potentially multidimensional political issues.

(1984), Osborne (1993) and Callander (2005) examine the impact of sequential decision making in political competitions.[5] One of the main questions posed is whether sequential entry induces policy divergence under the assumption that each candidate commits to his policy announcement.[6]

Our study departs from the literature in the following respects. First, the main focus of the paper is on the role of minor candidates, not whether policy divergence occurs. Our main result then sheds light on the substantial influence of minor candidates, a feature little investigated in the existing works. Second, our analysis clarifies the impact of sequential decision making in a framework where potential candidates cannot commit to any policy announcement during the campaign. To our best knowledge, the literature on citizen–candidate models focuses mainly on simultaneous entry.

These differences provide the following insights. First, our analysis reveals the weakness of a Condorcet winner. In the simultaneous entry framework, if a unique Condorcet winner exists, then an equilibrium always exists wherein the Condorcet winner is unopposed (Besley & Coate 1997). In contrast, that equilibrium may not exist in our sequential entry framework. Second, we demonstrate strategic candidacy on two-entrant competition: one of two candidates enters without any chance of winning.[7] In our model, the minor candidate attempts strategic candidacy to prevent the less preferred rival to enter. Although the empirical evidence suggests that strategic candidacy arises in two-candidate competition, the baseline models by Osborne & Slivinski (1996) and Besley & Coate (1997) do not explain this phenomenon clearly.[8] Asako (2015) and Ishihara (2016)

[5]Iwanami (2016) analyzes elections in UNSC, using a model in which a Tullock (1980) contest occurs following the sequential decision making of candidacy.

[6]Palfrey (1984) obtains a positive answer to this question under the essential assumption that the followers necessarily enter. Callander (2005) relaxes this demanding assumption by considering multidistrict competition, and successfully demonstrates policy divergence. While the order of decision making is exogenously fixed in these studies, Osborne (1993) considers a model where the order is determined endogenously. In his framework, the entrant is likely to choose the position of the median voter.

[7]Another body of work exists with a slightly different interest in that it investigates whether a voting procedure is immune to the threat of strategic candidacy. See Dutta et al. (2001, 2002), Ehlers & Weymark (2003), Eraslan & McLennan (2004), and Samejima (2007).

[8]For example, a fresh weak challenger is opposed to an established dominant incumbent. See Jewell & Breaux (1988, 1991) and Brady et al. (2007) for details.

successfully demonstrate strategic candidacy in two-candidate equilibria by introducing partial commitment to platforms and repeated interaction, respectively. This paper can be regarded as complementing those studies in that we provide another rationale for this phenomenon.[9]

The rest of the paper is organized as follows. Section 2 conducts a formal analysis of the sequential entry decision game. Section 3 concludes the paper. The Appendix A provides the proof of Theorem 1, the main result. In the Supplementary Appendix, we provide an analysis of the generalized model and all omitted proofs.

2 Political Competition with Sequential Entry

2.1 The Environment

In this section, we analyze sequential entry models wherein (i) there are three potential candidates, and (ii) the order of decision making is fixed exogenously.[10] The political competition proceeds according to the following three steps. First, each potential candidate decides sequentially whether to stand for election or not. Second, the voting procedure determines the winner from the set of the standing candidates. Finally, the winner implements a policy.

There exist three potential candidates (hereafter, *candidates*), and candidate $i \in \mathcal{N} \equiv \{1, 2, 3\}$ is the ith mover. Let $a_i \in A \equiv \{E, N\}$ be the action of candidate $i \in \mathcal{N}$, where E (respectively N) means entry (respectively no entry). Each candidate decides whether to stand for election by observing the decisions of the past candidates. We say that a candidate is an *entrant* if he stands for election.

[9] These earlier studies of sequential entry exclude the possibility of strategic candidacy mainly because of their assumptions on preferences. In Osborne (1993), each potential candidate prefers to exit than to lose at the post-entry voting stage and thus never chooses to run with no chance of winning in equilibrium. Callander (2005) and Iwanami (2016) assume that each potential candidate is office motivated, which guarantees that each candidate stays out unless there is a positive probability of winning.

[10] In the Supplementary Appendix, we consider a model for which the order of decision making is endogenous.

After decision making on entry, the voting procedure determines the winner of the election. We assume that as in citizen–candidate frameworks, entrants cannot commit to a policy platform during the campaign, and the winner implements his ideal policy. Hence, the identities of the entrants fully characterize the electoral outcome. Let $S \subseteq \mathcal{N}$ be an arbitrary set of entrants. We take a reduced-form approach in the voting procedure as follows. Let $C : 2^{\mathcal{N}} \to \mathcal{N} \cup \{0\}$ be a voting function representing the winning entrant $C(S)$ given the set of entrants S, where we assume that (i) $C(S) \in S$ for any $S \in 2^{\mathcal{N}} \backslash \{\emptyset\}$, and (ii) $C(\emptyset) = 0$, where 0 is a status quo policy in the case of no entrants.[11][12]

When entrant $j \in S$ wins the election, candidate $i \in \mathcal{N}$ obtains political benefit $v^i(j)$. When a candidate stands for election, he incurs an entry cost $d > 0$. Then, given that entrant j wins the election, the payoff for candidate i is $v^i(j) - d$ if he stands for election and $v^i(j)$ if he does not. We assume the following properties on $v^i(\cdot)$.

Assumption 1 *Candidate i's political benefit $v^i(\cdot)$ satisfies:*

1. *$v^i(j) \neq v^i(k)$ and $v^i(j) - d \neq v^i(k)$ for any $i \in \mathcal{N}$ and $j, k \in \mathcal{N} \cup \{0\}$ with $j \neq k$,*

2. *$v^i(i) - d > v^i(j)$ for any $i, j \in \mathcal{N}$ with $i \neq j$.*

The first assumption ensures that candidates have strict preferences over outcomes, and the second means that each candidate wants to be the first-place-vote winner.

We define three kinds of candidates, the *minor*, the *Condorcet winner*, and the *grand winner* as follows, referred to as M, CW, and GW, respectively.

Definition 1 1. *Candidate i is M if $i \neq C(S)$ for any $S \in 2^{\mathcal{N}} \backslash \{i\}$.*

2. *Candidate i is CW if $C(\{i, j\}) = i$ for any $j \in \mathcal{N} \backslash \{i\}$.*

3. *Candidate i is GW if $C(\mathcal{N}) = i$.*

[11] The status quo policy never emerges in equilibrium under the assumptions made below. The analysis can be extended to the case in which a tie is allowed. See the Supplementary Appendix.

[12] This approach obviously covers the cases of sincere voters. Even if the voters are strategic, the voting outcome can be summarized by the voting function as long as the voter's strategy depends only on the set of the candidates.

Candidate M is weak in that he cannot win as long as there is a rival in the vote although he may split votes non-negligibly; CW is a candidate who wins in any pairwise vote, and GW is a candidate who wins if all candidates stand for election. Throughout the paper, we assume the existence of a minor candidate.

Assumption 2 *There exists a unique M.*

By definition, a unique GW always exists. Furthermore, Assumption 2 guarantees that a unique CW likewise always exists. Note that CW and GW could coincide.

We denote candidate i's strategy by σ_i, and we are interested in a subgame perfect equilibrium (hereafter, SPE).[13] For a strategy profile $\sigma \equiv (\sigma_1, \sigma_2, \sigma_3)$, let $\hat{S}(\sigma)$ be the set of the entrants under the associated play. We define the notion of *kingmaker* as follows.

Definition 2 *Candidate i is a kingmaker under voting procedure $C(\cdot)$ if for any preference satisfying Assumption 1, $v^i(j) > v^i(k)$ for $j, k \in \mathcal{N} \backslash \{i\}$ and $j \neq k$ implies $j = C(\hat{S}(\sigma))$.*

In words, if candidate i is a kingmaker, then for any preference of candidate i, the other candidate whom he prefers to win always wins.

2.2 Analysis

First, we show the following proposition on the property of kingmakers.

Proposition 1 *1. If a kingmaker exists, then he must be M.*

2. If $CW = GW$, then $\hat{S}(\sigma) = \{CW\} = \{GW\}$ for any SPE σ, so no kingmaker exists.

Proposition 1 implies that when we look for candidates who are kingmakers, we focus on the unique M. Nevertheless, M does not necessarily influence the electoral outcome as

[13] The formal definition of the strategy is found in Appendix A.

a kingmaker. If CW coincides with GW, then that candidate is sufficiently strong and always wins regardless of other candidates' preferences. In this case, no room for strategic manipulation by others opens up. However, in any other case, M must be a kingmaker.

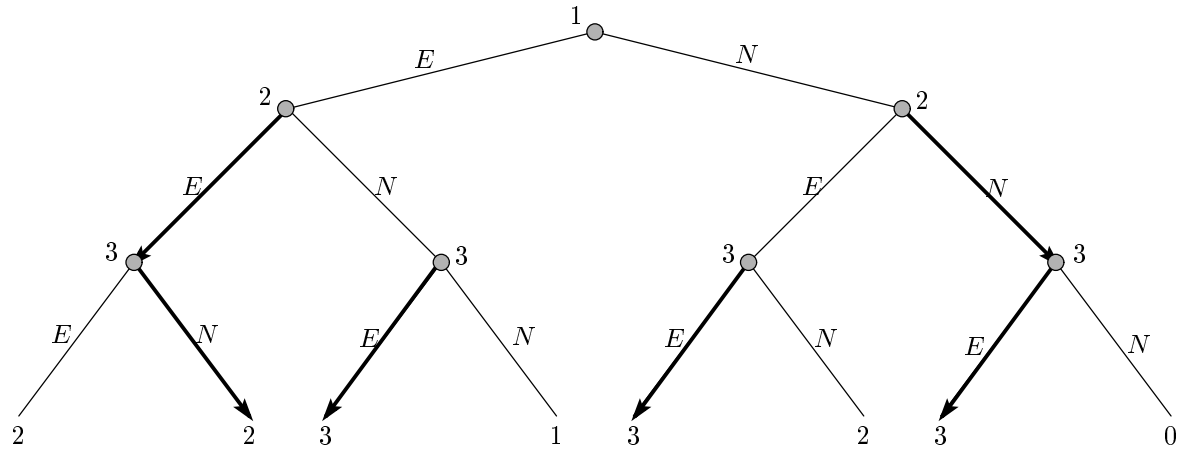
Theorem 1 *As long as $CW \neq GW$, M is a kingmaker under any voting procedure.*

Theorem 1 has an implication for the strength of CW, which is different from canonical citizen–candidate models with simultaneous entry decisions. Specifically, Besley & Coate (1997,Corollary 1) show that if a CW exists as in our model, then there always exists an equilibrium such that the CW is unopposed (and consequently becomes the winner). However, our theorem shows that if the entry decision is sequential, then, depending on the minor candidate's preference, an equilibrium in which CW wins may not exist. In this sense, our analysis demonstrates a weakness of CW that is not mentioned in the previous literature.

Although M is always a kingmaker regardless of the order of decision making and the details of the voting procedure as long as $CW \neq GW$, *how* M affects the electoral outcome is influenced by these factors. Specifically, the withdrawal of M's entry enables CW to win. In contrast, for GW to win, M either *attempts strategic candidacy* or *threatens* CW by monitoring CW's decision in light of the two factors just mentioned. The following two examples illustrate what "strategic candidacy" and "threatening" mean, and how M influences the electoral outcome.

2.2.1 Strategic Candidacy

We consider a situation in which M makes a decision before CW makes his. As Example 1, suppose that $C(\{1,2\}) = 2$, $C(\{1,3\}) = 3$, $C(\{2,3\}) = 3$ and $C(\mathcal{N}) = 2$. These voting outcomes imply that candidate 1, 2 and 3 are M, GW and CW, respectively. The assumption that each candidate wants to win the election in the first place implies that the best responses of candidates 2 and 3 are fully characterized by backward induction.



(The terminal nodes represent the winner)

Figure 1: Decision Procedure when $M = 1$, $CW = 3$ and $GW = 2$

Figure 1 illustrates the decision procedure and the best responses of candidates 2 and 3. Accordingly, candidate 2 wins if candidate 1 enters while candidate 3 wins if candidate 1 does not.

The preference of M then completely determines the winner. If $v^1(2) < v^1(3)$, that is, M prefers CW to GW, then candidate 1 stays out of the race to ensure that candidate 3 wins. Because of candidate 1's exit, a three-entrant competition never occurs. Hence, CW becomes the winner. On the other hand, if $v^1(2) > v^1(3)$, that is, M prefers GW to CW, then candidate 1 enters to enable candidate 2 to win. Once candidate 1 enters, it is impossible for CW to face a two-entrant competition because candidate 1's entry causes the entry of GW. It is favorable for candidate 1 to induce GW to win by entry, even if entry is costly and candidate 1 himself has no chance of winning.

We define strategic candidacy as entering without any chance of winning. Formally, we say that strategic candidacy occurs in strategy profile σ if $\{C(\hat{S}(\sigma))\} \neq \hat{S}(\sigma)$, and candidate i attempts strategic candidacy in strategy profile σ if $i \in \hat{S}(\sigma) \backslash \{C(\hat{S}(\sigma))\}$. The following theorem states that strategic candidacy occurs when M decides before CW and prefers GW to be the election's winner.

Theorem 2 *Strategic candidacy occurs in SPE if and only if the following conditions hold: (a) M makes a decision before CW, and (b) $v^M(CW) < v^M(GW)$. Furthermore,*

if strategic candidacy occurs in SPE σ, then M is the candidate attempting strategic candidacy.

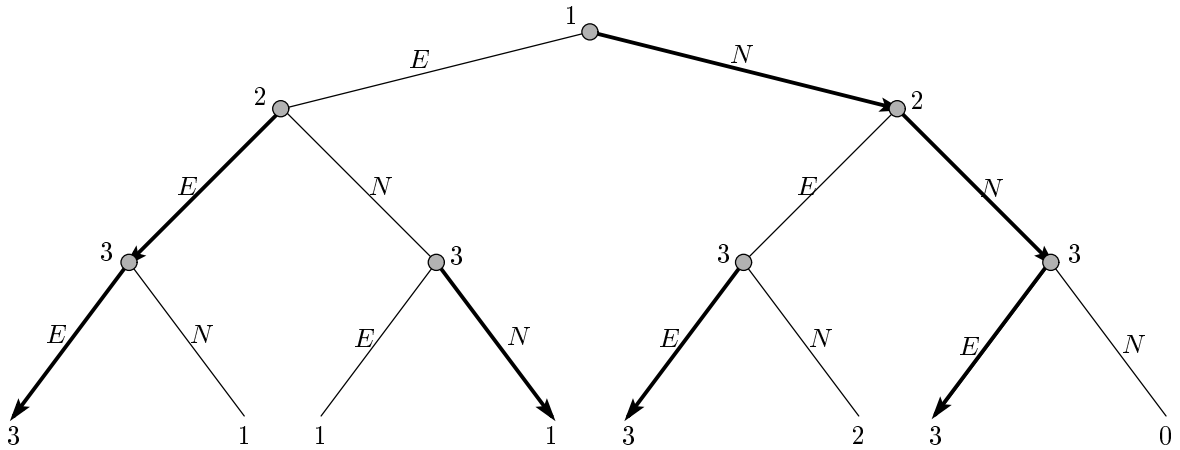
In sequential entry games, the effect of strategic candidacy is somewhat different from that of the simultaneous entry games in the literature. The rationale for strategic candidacy is changing the election's winner. To guarantee this outcome in simultaneous entry games, at least three candidates must stand for election.[14] In contrast, in a sequential entry game, strategic candidacy may emerge even in two-candidate equilibrium. Intuitively, in our model, strategic candidacy can work as a leader's commitment to entry, which deters a follower's entry. As shown in Example 1, M's commitment to enter makes it difficult to support a scenario in which CW wins, as it deters entry by CW. In other words, this commitment to enter can crowd CW out, and cause GW to win. In the simultaneous entry scenario, such strategic candidacy never occurs: given that M predicts that he and GW are the entrants in equilibrium, M strictly prefers to exit because he can avoid the entry cost without changing the electoral outcome.

Strategic candidacy is an option allowing M to impose his preference on the electoral outcome when he moves before CW. When M is the follower of CW, M influences the election by threatening CW, as shown in the next example.

2.2.2 Threatening

In Example 2, M decides after CW does. Specifically, suppose that $C(\{1,2\}) = 1$, $C(\{1,3\}) = 1$, $C(\{2,3\}) = 3$ and $C(\mathcal{N}) = 3$. These voting outcomes imply that candidates 1, 2 and 3 are CW, M and GW, respectively. Furthermore, we assume that M prefers GW to CW, that is, $v^2(3) > v^2(1)$. Figure 2 illustrates the decision procedure and the SPE, and the winner is candidate 3, whom M prefers to the other rival.

[14] This result relies on the assumption that the winner cannot credibly choose a policy different from his ideal point. When the winner can credibly implement a policy other than his ideal point, strategic candidacy may arise in a two-candidate equilibrium because entry may change the policy implemented by the rival (Asako 2015, Ishihara 2016).



(The terminal nodes represent the winner)

Figure 2: Decision Procedure when $M = 2$, $CW = 1$, $GW = 3$ and $v^2(3) > v^2(1)$

Here, M "threatens" CW's decision in the sense that M's decision depends on whether CW enters, which causes GW to win. On the one hand, after observing candidate 1's entry, candidate 2 can credibly enter with the threat of inducing a three-entrant election. Expecting candidate 2's entry, candidate 1 surrenders because he cannot win the three-entrant election. On the other hand, after observing candidate 1's exit, M now has no incentive to enter because candidate 3 wins regardless of M's decision. As a result, candidate 3 becomes the unopposed winner in equilibrium.

This logic is generally valid when M is the follower of CW. Provided that M makes an entry decision after candidate i makes his, we say that M *threatens* candidate i in strategy profile σ if, in the subgame starting from candidate i that is reached under σ, M chooses the same action as candidate i.[15] By threatening CW, M can successfully crowd out CW without entry costs, which makes GW the unopposed winner, as shown in the following theorem.

Theorem 3 *Suppose that M makes a decision after CW makes his. Then, M threatens CW in SPE if and only if $v^M(CW) < v^M(GW)$. Furthermore, if M threatens CW in SPE σ, then $\hat{S}(\sigma) = \{GW\}$.*

[15]The formal definition of threatening is found in the Supplementary Appendix.

3 Concluding Remarks

This article studied a sequential entry model with three potential candidates who cannot commit credibly to any policy announcement, and highlighted the impact of minor candidates. We demonstrated that in sequential entry games, the minor candidate behaves as a kingmaker under any voting procedure independent of the order of decision making unless the Condorcet winner coincides with the grand winner. To reflect his preference in the electoral outcome, the minor candidate either attempts strategic candidacy, threatens the Condorcet winner, or simply exits the election depending on the order of decision making, the voting procedure, and his or her own preference about whether the Condorcet winner or the grand winner is the election's winner.

As a final remark, we discuss the robustness of our results. First, the assumption of the exogenous decision-making order can be relaxed. In the Supplementary Appendix, we consider another model wherein the order of decision making is determined endogenously, and our main insights still hold. We admit that the assumption of three potential candidates is essential. Adding an extra candidate who cannot affect the voting outcome at all does not change our results. Nevertheless, if the additional candidate does affect the outcome, then the election results would depend materially on the order of decision making, candidates' preferences, and voting procedures. This makes it more difficult to induce a general property in electoral outcomes in sequential entry models. We defer that issue to future research.

A Appendix: Proof of Theorem 1

A.1 Strategies

We represent political competition as an extensive-form game in which each candidate makes an entry decision. Let $h_i \in \{\emptyset, \{1\}, \{2\}, \{1, 2\}\}$ be the set of entrants who have

decided to enter when candidate i makes a decision (hereafter, history).[16] Denote candidate i's strategy by $\sigma_i(h_i) \in \{E, N\}$, meaning the action undertaken by candidate i after history h_i. Let $\hat{h}_i(\sigma)$ be candidate i's history on the equilibrium path under strategy profile $\sigma \equiv (\sigma_1, \sigma_2, \sigma_3)$.

A.2 Proof of Theorem 1

A.2.1 Lemmas for the Proof

We first provide several lemmas to show the theorem.[17]

Lemma 1 *There exists no SPE σ such that $M = C(\hat{S}(\sigma))$.*

Lemma 2 *There exists no SPE σ such that either (i) $\hat{S}(\sigma) = \emptyset$ or (ii) $\hat{S}(\sigma) = \{M, CW\}$.*

Lemma 3 *Candidates other than M do not attempt strategic candidacy in equilibrium.*

Lemma 4 *If M attempts strategic candidacy in equilibrium, then $CW \neq GW$ and $v^M(CW) < v^M(GW)$.*

Lemma 5 *Suppose that $CW \neq GW$. If $CW \neq 1$ and $v^M(CW) < v^M(GW)$, then $\sigma_{CW}(\hat{h}_{CW}(E, \sigma_{-1})) = N$ for any SPE σ.*

A.2.2 Proof of Theorem 1

By Lemma 1, $C(\hat{S}(\sigma)) = CW$ or GW. Then, it is enough to show that $C(\hat{S}(\sigma)) = CW$ if $v^M(CW) > v^M(GW)$ and $C(\hat{S}(\sigma)) = GW$ if $v^M(CW) < v^M(GW)$.

Suppose that $C(\hat{S}(\sigma)) = CW$ when $v^M(CW) < v^M(GW)$. Then, by Lemmas 2 and 3, $\hat{S}(\sigma) = \{CW\}$, which implies that $\sigma_{CW}(\hat{h}_{CW}(\sigma)) = E$. Note also that because of $v^M(CW) < v^M(GW)$, Assumption 1 guarantees $v^k(CW) < v^k(GW) - d$ for any $k \in \{GW, M\}$. We now show that in the following two cases, there is a candidate who has an incentive to deviate.

[16] Strictly, the history must satisfy $h_1 = \emptyset$, $h_2 \in \{\emptyset, \{1\}\}$, and $h_3 \in \{\emptyset, \{1\}, \{2\}, \{1, 2\}\}$.

[17] The lemmas here are implied by other lemmas displayed in the Supplementary Appendix.

1. Suppose that $CW = 1$. Note that $\{GW, M\} = \{2, 3\}$, $\sigma_2(\hat{h}_2(\sigma)) = N$, and candidate 2's equilibrium payoff is $v^2(CW)$. Now, suppose that candidate 2 deviates to $a_2 = E$ at history $\hat{h}_2(\sigma)$. Given history $h'_3 = \{1, 2\}$, because $v^3(CW) < v^3(GW) - d$, candidate 3 chooses to enter and induces GW to be the winner, rather than choosing to stay out and inducing CW to be the winner. Hence, candidate 2's payoff from the deviation is $v^2(GW) - d$, and he has an incentive to deviate.

2. Suppose that $CW \neq 1$. Note that $\sigma_1 = N$, and candidate 1's equilibrium payoff is $v^1(CW)$. Now, suppose that candidate 1 deviates to $a_1 = E$. By Lemma 5, $\sigma_{CW}(\hat{h}_{CW}(E, \sigma_{-1})) = N$. Then, the winner is either M or GW. When $M = 1$, candidate 1's payoff from this deviation is $v^1(M) - d$ or $v^1(GW) - d$, both of which are greater than $v^1(CW)$ because $v^1(CW) < v^1(GW) - d < v^1(M) - d$. When $GW = 1$, because GW enters and CW does not enter, this deviation induces GW to be the winner, which yields candidate 1's payoff $v^1(GW) - d$ greater than $v^1(CW)$. Hence, candidate 1 has an incentive to deviate.

Suppose that $C(\hat{S}(\sigma)) = GW$ when $v^M(CW) > v^M(GW)$. Then, by Lemma 4, $\hat{S}(\sigma) = \{GW\}$. Note that $\sigma_{CW}(\hat{h}_{CW}(\sigma)) = N$, and then CW's equilibrium payoff is $v^{CW}(GW)$. Now, we show that CW has an incentive to deviate to $a_{CW} = E$ at history $\hat{h}_{CW}(\sigma)$.

1. Suppose that CW is a leader of M. Consider M's decision at history after CW's deviation. If $a_M = E$, the winner is either CW or GW. Hence, M's payoff is either $v^M(CW) - d$ or $v^M(GW) - d$, respectively. If $a_M = N$, then CW wins. Hence, M's payoff is $v^M(CW)$. Because $v^M(CW) > v^M(GW)$, M chooses $a_M = N$ after CW's entry. Given M's behavior, CW's deviation to $a_{CW} = E$ gives CW payoff $v^{CW}(CW) - d$, which is greater than $v^{CW}(GW)$.

2. Suppose that CW is a follower of M. Because $\hat{S}(\sigma) = \{GW\}$, $M \notin \hat{h}_{CW}(\sigma)$. Hence,

CW's deviation makes him become the winner, and then his payoff is $v^{CW}(CW)-d$, which is greater than $v^{CW}(GW)$.

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B Supplementary Appendix

B.1 Multiple Winners

The baseline model focuses on cases in which $\#(C(S)) = 1$ for any $S \in 2^{\mathcal{N}} \setminus \{\emptyset\}$, that is, there exists a unique winner for any set of entrants. This section relaxes this assumption and demonstrates that the results are qualitatively the same. This section also provides all the proofs of the results.

B.1.1 Modified Definitions and Assumptions

Now, C is defined as a correspondence instead of a function. Let $C : 2^{\mathcal{N}} \to (2^{\mathcal{N}} \setminus \{\emptyset\}) \cup \{0\}$ be a voting correspondence representing the set of winners $C(S)$, given the set of entrants S, where we assume (i) $C(S) \subseteq S$ for any $S \in 2^{\mathcal{N}} \setminus \{\emptyset\}$, and (ii) $C(\emptyset) = \{0\}$. We assume that if there are two or more winners in a vote, then each of them randomly wins with equal probability $1/\#(C(S))$.[18]

We modify the definition of the minor, the Condorcet winner, and the grand winner as follows.

Definition 3 *1. Candidate i is a minor if $i \notin C(S)$ for any $S \in 2^{\mathcal{N}} \setminus \{i\}$.*

2. Candidate i is a Condorcet winner if $i \in C(\{i, j\})$ for any $j \in \mathcal{N} \setminus \{i\}$. Candidate i is a strict Condorcet winner if $C(\{i, j\}) = \{i\}$ for any $j \in \mathcal{N} \setminus \{i\}$.

3. Candidate i is a grand winner if $i \in C(\mathcal{N})$. Candidate i is a strict grand winner if $C(\mathcal{N}) = \{i\}$.

A Condorcet (resp. grand) winner is now a candidate who wins *with positive probability* in any pairwise vote (resp. a three-entrant competition). A strict Condorcet or grand winner wins certainly in the corresponding vote, which is the same as the definition of

[18] The assumption of the tie-breaking rule can be relaxed as long as the entry cost is sufficiently small.

the Condorcet or grand winner in the baseline model. It should be mentioned that if a strict Condorcet (resp. grand) winner exists, then the Condorcet (resp. grand) winner is unique.

We maintain Assumption 2 so that a minor candidate uniquely exists. Similar to the baseline model, a grand winner and a Condorcet winner always exist under Assumption 2. Nevertheless, there might be multiple Condorcet winners or multiple grand winners. Now, CW and GW denote the *set* of the Condorcet winners and the grand winners, respectively. Note that the baseline model is a special case such that $\#(CW) = \#(GW) = 1$. We maintain M to be the minor *candidate*, not the set of the minor candidates.

Each candidate is interested in maximizing the expected payoff. In addition to Assumption 1, we suppose the following.

Assumption 3 *$|v^i(j) - v^i(k)|/3 > d$ for any $i \in \mathcal{N}$ and $j, k \in \mathcal{N} \cup \{0\}$ with $j \neq k$.*

This additional assumption guarantees that the entry cost d is so small that entry is beneficial as long as the electoral outcome can improve even though the entrant has no chance of winning. Note that when $\#(CW) = \#(GW) = 1$, this additional assumption is innocuous since there is no uncertainty on the voting outcome.

We also modify the notion of kingmaker as follows.

Definition 4 *Candidate i is a kingmaker under voting procedure $C(\cdot)$ if for any preference satisfying Assumptions 1 and 3, $v^i(j) > v^i(k)$ for $j, k \in \mathcal{N} \backslash \{i\}$ and $j \neq k$ implies $j \in C(\hat{S}(\sigma))$.*

If candidate i is a kingmaker, then for any preference, he can guarantee his preferred candidate to win *with positive probability.* Note that this definition coincides with Definition 2 when $\#(CW) = \#(GW) = 1$.

The strategy is defined as in Appendix A.1. We say that strategic candidacy occurs in strategy profile σ if $C(\hat{S}(\sigma)) \neq \hat{S}(\sigma)$, and candidate i attempts strategic candidacy in

strategy profile σ if $i \in \hat{S}(\sigma) \backslash C(\hat{S}(\sigma))$. In order to define threatening, given candidate i's strategy σ_i and history h_i, let $\hat{\sigma}_i(a_i, h_i \mid \sigma_i)$ be candidate i's strategy such that $a_i \in \{E, N\}$ is chosen at history h_i and $\sigma_i(\tilde{h}_i)$ is chosen at any other history $\tilde{h}_i \neq h_i$. Given that candidate i makes a decision before M, we say that M threatens candidate i in strategy profile σ if $\sigma_M(\hat{h}_M(\hat{\sigma}_i(a_i, \hat{h}_i(\sigma) \mid \sigma_i), \sigma_{-i})) = a_i$ for both $a_i \in \{E, N\}$. In words, $\hat{\sigma}_i(a_i, \hat{h}_i(\sigma) \mid \sigma_i)$ is candidate i's modified strategy such that he chooses a_i at the node reached under σ. Then, $\hat{h}_M(\hat{\sigma}_i(a_i, \hat{h}_i(\sigma) \mid \sigma_i), \sigma_{-i})$ is M's history consistent with strategy profile $(\hat{\sigma}_i, \sigma_{-i})$. When M threatens candidate i, M coordinates with candidate i's action given that history $\hat{h}_i(\sigma)$ has been realized. Obviously, if M threatens candidate i, then they must take the same action on the equilibrium path.

B.1.2 Main Results

When we generalize the definition of CW and GW as the sets, the statements are established as follows.

Proposition 2 *1. If there exists a kingmaker, then he must M.*

2. If $CW = GW$, then $\hat{S}(\sigma) = CW = GW$ for any SPE σ.

Theorem 4 *As long as $CW \neq GW$, M is a kingmaker under any voting procedure.*

Theorem 5 *Suppose that $i \in CW$ and $GW = \{j\}$ with $j \neq i$. Then, strategic candidacy occurs in equilibrium if and only if the following conditions hold: (a) M makes a decision before candidate i, and (b) $v^M(i) < v^M(j)$. Furthermore, if strategic candidacy occurs in SPE σ, then $\hat{S}(\sigma) = \{M, j\}$ holds.*

Theorem 6 *Suppose that $CW = \{i\}$ and $GW = \{i, j\}$. Then, strategic candidacy occurs in equilibrium if and only if $v^M(i) < v^M(j)$. Furthermore, if strategic candidacy occurs in SPE σ, then $\hat{S}(\sigma) = \mathcal{N}$.*

	M is a leader of i	M is a follower of i
$CW = \{i\}, GW = \{j\}$	Strategic Candidacy	Threatening
$CW = \{i, j\}, GW = \{j\}$	Strategic Candidacy	Threatening
$CW = \{i\}, GW = \{i, j\}$	Strategic Candidacy	Strategic Candidacy

Table 1: Minor's Behavior when He Prefers a Grand Winner (Candidate j)

Theorem 7 *Suppose that $i \in CW$ and $GW = \{j\}$ with $j \neq i$, and M makes a decision after candidate i. Then, M threatens candidate i in equilibrium if and only if $v^M(i) < v^M(j)$. Furthermore, if M threatens candidate i in SPE σ, then $\hat{S}(\sigma) = \{j\}$.*

Proposition 1 and Theorems 1, 2, and 3 in the body of the paper are implied by Proposition 2 and Theorems 4, 5, and 7, respectively. Our arguments in the baseline model are still valid. Independent of the order of decision making and the voting procedure, M's preferred rival always wins with positive probability in any equilibrium unless $CW = GW$. If M prefers a Condorcet winner, then he simply stays out, by which he can deter a three-entrant vote and the Condorcet winner wins with positive probability. Conversely, if M prefers a grand winner, M's behavior is summarized in Table 1. If the grand winner is unique, then M either attempts strategic candidacy or threatens the other rival (candidate i in Table 1) depending on whether M makes a decision before or after candidate i. Theorem 6 implies an exception when there are two grand winners. If M prefers the rival who is not the Condorcet winner, then strategic candidacy must occur regardless of the order of decision making.

B.1.3 Preliminary Results and Proof of Proposition 2

Hereafter, let $\mathcal{N} \equiv \{i, j, M\}$. We first show a series of lemmas and Proposition 2 as follows.

Lemma 6 *Let l and f be the leader and the follower between candidate i and j, respectively. If $l \notin h_f$, then $\sigma_f(h_f) = E$ for any SPE σ.*

Proof of Lemma 6 Suppose, in contrast, that there exists SPE σ' in which there exists history h'_f such that $l \notin h'_f$ and $\sigma'_f(h'_f) = N$. Note that candidate f's payoff at the subgame starting from history h'_f is either $v^f(M)$ or $v^f(0)$. Now, suppose that candidate f deviates to $a_f = E$ at history h'_f. Because $l \notin h'_f$, candidate f wins for certain in the vote whatever M's decision. Then, candidate f's payoff from this deviation is $v^f(f) - d$, which is greater than $v^f(M)$ and $v^f(0)$ by Assumption 1. Therefore candidate f has an incentive to deviate. ■

Lemma 7 *There exists no SPE σ such that $M \in C(\hat{S}(\sigma))$.*

Proof of Lemma 7 Suppose, in contrast, that there exists SPE σ such that $M \in C(\hat{S}(\sigma))$. By definition of M, $\hat{S}(\sigma) = \{M\}$ should hold, which implies $C(\hat{S}(\sigma)) = \{M\}$. There are two cases to be checked.

1. Suppose that either $M = 1$ or 2. Note that $\sigma_3(\hat{h}_3(\sigma)) = N$, and $i \notin \hat{h}_3(\sigma)$ where $i \in \{1, 2\} \backslash \{M\}$. However, this is a contradiction by Lemma 6.

2. Suppose that $M = 3$. By the hypothesis, $1 \notin \hat{h}_2(\sigma)$ and $\sigma_2(\hat{h}_2(\sigma)) = N$. However, this is a contradiction by Lemma 6. ■

Lemma 8 *If $i \in CW \cap GW$, then $\sigma_i(h_i) = E$ for any history h_i in any SPE σ.*

Proof of Lemma 8 Suppose, in contrast, that there exists SPE σ' in which there exists history h'_i such that $\sigma'_i(h'_i) = N$. If $\sigma'_i(h'_i) = N$, the set of the winner is either $\{j\}$, $\{M\}$, or $\{0\}$. If, on the other hand, candidate i chooses E at history h'_i, then because candidate i must win and M never wins, the set of the winner is either $\{i, j\}$ or $\{i\}$. Hence, σ' is an SPE only if, the following conditions are satisfied:

- if candidate i enters at history h'_i, then the set of the winners after h'_i is $\{i, j\}$; and

- if candidate i exits at history h'_i, then the set of the winners after h'_i is $\{M\}$ or $\{0\}$.

Otherwise, candidate i has an incentive to enter at history h_i' under Assumptions 1 and 3. There are the following cases to be checked.

1. Suppose that candidate i is a follower of candidate j. Because the set of the winners is either $\{M\}$ or $\{0\}$ given that candidate i exits, $j \notin h_i'$, which contradicts the fact that the set of the winners is $\{i, j\}$ given that candidate i enters.

2. Suppose that candidate i is a leader of plyaer j. Because $\sigma_i'(h_i') = N$, candidate j enters at the subgame starting from history h_i' by Lemma 6. However, it contradicts the fact that the set of the winners must be $\{M\}$ or $\{0\}$. ■

Proof of Proposition 2

1. Suppose, in contrast, that candidate $i \neq M$ is a kingmaker. Then, by the definition of kingmaker, provided that candidate i prefers M to the other, there exists an SPE σ such that $M \in C(\hat{S}(\sigma))$. However, this contradicts Lemma 7.

2. There are two cases to be considered.

1. Suppose that $\#(CW) = 1$. Let $CW \equiv \{i\}$ and $k \in \mathcal{N} \backslash \{i\}$. By Lemma 8, $\sigma_i(h_i) = E$ for any history h_i and SPE σ. Because candidate i is both a strict Condorcet winner and a strict grand winner, candidate i wins for certain as long as $\sigma_i(h_i) = E$ for any history h_i, independent of candidate k's decision. Hence, to save the entry cost, $\sigma_k(\hat{h}_k(\sigma)) = N$ for any $k \in \mathcal{N} \backslash \{i\}$. Thus, $\hat{S}(\sigma) = \{i\} = CW$.

2. Suppose that $\#(CW) = 2$. Let $CW \equiv \{i, j\}$, and then $\mathcal{N} \backslash CW = \{M\}$. By Lemma 8, $\sigma_i(h_i) = E$ and $\sigma_j(h_j) = E$ for any history h_i, h_j, and SPE σ. Note that the winners are candidates i and j independent of M's behavior. Then, M never enters to save the entry cost. Thus, $\hat{S}(\sigma) = \{i, j\} = CW$. ■

Lemma 9 *Suppose that $CW = \{i\}$, $GW = \{i, j\}$, and $v^M(i) < v^M(j)$ hold. Let l and f be the leader and the follower between candidates j and M, respectively. If $l \in h_f$, then $\sigma_f(h_f) = E$ for any SPE σ.*

Proof of Lemma 9 Suppose, in contrast, that there exists SPE σ and history h'_f such that $l \in h'_f$ and $\sigma_f(h'_f) = N$. By Lemma 8, $\sigma_i(h_i) = E$ for any history h_i. Then, after history h'_f, the set of the entrants is $S' = \{i, l\}$. Because $C(S') = \{i\}$, candidate f's payoff is $v^f(i)$. Now, suppose that candidate f deviates to $a_f = E$ at history h'_f. Then, at the terminal node, the set of the entrants becomes $S'' = \mathcal{N}$. Because $C(S'') = \{i, j\}$, candidate f's expected payoff is $\sum_{k \in GW} v^f(k)/2 - d$. However, Assumption 3 and $v^M(i) < v^M(j)$ guarantee that $\sum_{k \in GW} v^f(k)/2 - d > v^f(i)$ for any $f \in \{M, j\}$. Therefore, candidate f has an incentive to deviate. ■

Lemma 10 *If there exists SPE σ such that $\hat{S}(\sigma) = \mathcal{N}$ under voting procedure $C(\cdot)$, then $C(\cdot)$ must satisfy $\#(CW) = 1$ and $\#(GW) = 2$.*

Proof of Lemma 10 Suppose, in contrast, that there exists SPE σ such that $\hat{S}(\sigma) = \mathcal{N}$ when either $\#(CW) \neq 1$ or $\#(GW) \neq 2$ holds. There are the following three cases to be checked.

1. Suppose that $CW = GW$. By Proposition 2, $\hat{S}(\sigma) = CW = GW \neq \mathcal{N}$, which is a contradiction.

2. Suppose that $CW = \{i\}$ and $GW = \{j\}$ with $i \neq j$. Because $\hat{S}(\sigma) = \mathcal{N}$, $\sigma_i(\hat{h}_i(\sigma)) = E$ at history $\hat{h}_i(\sigma)$. Hence, candidate i's payoff is $v^i(j) - d$. Now, suppose that candidate i deviates to $a_i = N$ at history $\hat{h}_i(\sigma)$. If candidate i is a leader of candidate j, then candidate j always enters by Lemma 6, and then he wins certainly. If candidate i is a follower of candidate j, then candidate j has already entered

and wins regardless of M's decision. Then, in both cases, candidate i's payoff is $v^i(j) > v^i(j) - d$, implying that candidate i has an incentive to deviate.

3. Suppose that $CW = \{i, j\}$ and $GW = \{j\}$. Note that $\sigma_i(\hat{h}_i(\sigma)) = E$ and candidate i's payoff is $v^i(j) - d$. If he deviates to $a_i = N$ at history $\hat{h}_i(\sigma)$, then the winner should be candidate j for certain because he always enters whatever the history is by Lemma 8. Then, candidate i's payoff is $v^i(j) > v^i(j) - d$, and he has an incentive to deviate. ■

Lemma 11 *If $CW = \{i\}$, $GW = \{i, j\}$, and $v^M(i) < v^M(j)$, then $\hat{S}(\sigma) = \mathcal{N}$ for any SPE σ.*

Proof of Lemma 11 Suppose that $CW = \{i\}$, $GW = \{i, j\}$, and $v^M(i) < v^M(j)$, and consider an arbitrary SPE σ. By Lemma 8, $\sigma_i(h_i) = E$ holds at any history h_i. Let l and f be the leader and the follower between candidates j and M. By Lemma 9, if $l \in h_f$, then $\sigma_f(h_f) = E$. Hence it is sufficient to show that $\sigma_l(\hat{h}_l(\sigma)) = E$. Now, we consider candidate l's decision at history $\hat{h}_l(\sigma)$. If $a_l = E$, then candidate l's expected payoff is $\left(v^l(i) + v^l(j)\right)/2 - d$ because $C(\mathcal{N}) = \{i, j\}$. If $a_l = N$, then his payoff is $v^l(i)$ because three-entrant competition never occurs. By Assumptions 1 and 3 and $v^M(j) > v^M(i)$ guarantee that $\left(v^l(i) + v^l(j)\right)/2 - d > v^l(i)$ for any $l \in \{M, j\}$. Thus, candidate l prefers to enter and then $\sigma_l(\hat{h}_l(\sigma)) = E$. ■

Lemma 12 *If there exists SPE σ such that $\hat{S}(\sigma) = \mathcal{N}$, then $CW = \{i\}$, $GW = \{i, j\}$, and $v^M(i) < v^M(j)$.*

Proof of Lemma 12 Suppose that there exists SPE σ such that $\hat{S}(\sigma) = \mathcal{N}$. By Lemma 10, the first and second conditions must hold. Suppose that $v^M(i) > v^M(j)$. Because $\hat{S}(\sigma) = \mathcal{N}$, $\sigma_M(\hat{h}_M(\sigma)) = E$. Hence, M's equilibrium expected payoff is $\left(v^M(i) + v^M(j)\right)/2 - d$. Now, suppose that M deviates to $a_M = N$ at history $\hat{h}_M(\sigma)$.

From Lemma 8, candidate i enters at any history. Because candidate i is a strict Condorcet winner and there are at most two entrants, candidate i wins for certain. Then, M's payoff from this deviation is $v^M(i)$. Because of Assumptions 1 and 3 and $v^M(i) > v^M(j)$, we have $v^M(i) > \left(v^M(i) + v^M(j)\right)/2 - d$. Then, M has an incentive to deviate at history $\hat{h}_M(\sigma)$, which is a contradiction. Therefore, $v^M(i) < v^M(j)$. ■

Lemma 13 *There exists no SPE σ such that either (i) $\hat{S}(\sigma) = \emptyset$ or (ii) $\hat{S}(\sigma) = \{M, i\}$ where candidate i is a strict Condorcet winner.*

Proof of Lemma 13 If an SPE σ satisfies $\hat{S}(\sigma) = \emptyset$, then candidate 3 obviously has an incentive to deviate to enter, by which he can be the unopposed winner. Next, suppose that there exists an SPE σ such that $\hat{S}(\sigma) = \{M, i\}$ where candidate i is a strict Condorcet winner. Note that $C(\hat{S}(\sigma)) = \{i\}$ and $\sigma_M(\hat{h}_M(\sigma)) = E$. Hence M's payoff is $v^M(i) - d$. Now, we show that if M deviates to $a_M = N$ at history $\hat{h}_M(\sigma)$, then his payoff is $v^M(i) > v^M(i) - d$ and he has an incentive to deviate. There are two cases to be checked.

1. Suppose that M is a follower of candidate i. Because $i \in \hat{h}_M(\sigma)$ and candidate i is a strict Condorcet winner, he wins for certain after M's exit regardless of the other candidate's decision. Hence, M's payoff after this deviation is $v^M(i)$.

2. Suppose that M is a leader of candidate i. Given $M \notin h_i$, candidate i enters because he wins regardless of the other candidate's decision. Hence, M's payoff is $v^M(i)$. ■

Lemma 14 *Suppose that $CW = \{i, j\}$, $GW = \{j\}$, $v^M(i) > v^M(j)$, and M is a follower of candidate i. If $i \in h$, then $\sigma_M(h) = N$ for any SPE σ.*

Proof of Lemma 14 Suppose $i \in h$. Note that candidate j always enters by Lemma 8. If M enters, then candidate j is the unique winner and M's payoff is $v^M(j) - d$. If M

exits instead, then the set of the winners is CW and M's payoff is $(v^M(j) + v^M(i))/2$, which is greater than $v^M(j) - d$ by Assumptions 1 and 3. ■

Lemma 15 *Candidates other than M do not attempt strategic candidacy in equilibrium.*

Proof of Lemma 15 Suppose, in contrast, that there exists an SPE σ such that candidate $i(\neq M)$ attempts strategic candidacy. There are three cases to be considered.

1. Suppose that $i \in CW$. Because candidate i loses in the vote, $\hat{S}(\sigma) = \mathcal{N}$ and $i \notin GW$. However, because $CW \subset GW$, by Lemma 12 and $\hat{S}(\sigma) = \mathcal{N}$, we must have $i \in GW$, which is a contradiction.

2. Suppose that $i \notin CW$ and $i \in GW$. Then, candidate $j \in CW$ is a strict Condorcet winner because $M \notin CW$. Because candidate i is a grand winner and loses in the vote, $\hat{S}(\sigma) = \{i, j\}$. Hence, candidate i's equilibrium payoff is $v^i(j) - d$. Now, suppose that candidate i deviates to $a_i = N$ at history $\hat{h}_i(\sigma)$. If candidate i is a leader of candidate j, then candidate j enters by Lemma 6, and wins regardless of M's decision. If candidate i is a follower of candidate j, then this deviation makes candidate j who has already entered win regardless of M's decision. Then, in both cases, candidate i's payoff from this deviation is $v^i(j) > v^i(j) - d$. Thus, candidate i has an incentive to deviate.

3. Suppose that $i \notin CW \cup GW$. Because candidate i is not M and $\#(\mathcal{N}) = 3$, $CW = GW = \{j\}$. However, by Proposition 2, $\hat{S}(\sigma) = CW = GW$, which contradicts $i \in \hat{S}(\sigma)$. ■

Lemma 16 *If M attempts strategic candidacy in equilibrium, then $CW \neq GW$ and $v^M(i) < v^M(j)$ where $i \in CW$ and $j \in GW$.*

Proof of Lemma 16 Suppose that there exists SPE σ such that $M \in \hat{S}(\sigma) \backslash C(\hat{S}(\sigma))$. If $CW = GW$, then by Proposition 2, $\hat{S}(\sigma) = CW = GW$, which contradicts $M \in \hat{S}(\sigma)$. Suppose next that $CW \neq GW$ and $v^M(i) > v^M(j)$. Because $M \in \hat{S}(\sigma)$ and $M \notin C(\hat{S}(\sigma))$, $\hat{S}(\sigma)$ is either $\{M, i\}$, $\{M, j\}$, or $\mathcal{N}$. Note that Lemma 12 implies $\hat{S}(\sigma) \neq \mathcal{N}$. If $\hat{S}(\sigma) = \{M, i\}$, then by Lemma 13, i is not a strict Condorcet winner, which implies $CW = \{i, j\}$ and $j \in CW \cap GW$. Hence, by Lemma 8, candidate j enters for any history h_j, which contradicts $\hat{S}(\sigma) = \{M, i\}$. Then $\hat{S}(\sigma) = \{M, j\}$ should hold and M's equilibrium payoff is $v^M(j) - d$. Now, consider that M deviates to $a_M = N$ at history $\hat{h}_M(\sigma)$. Because $v^M(M) > v^M(i) > v^M(j) > v^M(j) - d$, M prefers the deviation if there is a candidate who wins, i.e., the set of the entrants after the deviation, denoted by $\tilde{S}$, is not empty. If M is a follower of candidate j, then, because $\hat{S}(\sigma) = \{M, j\}$, at least candidate j is an entrant and then $\tilde{S} \neq \emptyset$. If M is a leader of candidate of j, then candidate 3 must enter given that no past candidates have entered. Then $\tilde{S} \neq \emptyset$. ■

Lemma 17 *Suppose that $CW = \{i\}$ and $GW = \{j\}$. If $i \neq 1$ and SPE σ satisfies $\sigma_i(\hat{h}_i(E, \sigma_{-1})) = E$, then $C(\hat{S}(E, \sigma_{-1})) = \{i\}$.*

Proof of Lemma 17 Suppose, in contrast, that $C(\hat{S}(E, \sigma_{-1})) \neq \{i\}$. Note that $\#(CW) = \#(GW) = 1$ implies that the set of the winners must be singleton in each terminal node. Because $\sigma_i(\hat{h}_i(E, \sigma_{-1})) = E$, we have $i \in \hat{S}(E, \sigma_{-1})$, which implies $C(\hat{S}(E, \sigma_{-1})) \neq \{M\}$. That is, $C(\hat{S}(E, \sigma_{-1})) = \{j\}$ should hold. Because $i \in \hat{S}(E, \sigma_{-1})$ and $C(\hat{S}(E, \sigma_{-1})) \neq \{i\}$, $\hat{S}(E, \sigma_{-1}) = \mathcal{N}$ must hold. Candidate i's payoff in strategy (E, σ_{-1}) is $v^i(j) - d$. Now, we show that candidate i prefers to deviate to $a_i = N$ at history $\hat{h}_i(E, \sigma_{-1})$. If candidate i is a leader of candidate j, then candidate j enters by Lemma 6 after exit of candidate i, and candidate j wins for certain. If candidate i is a follower of candidate j, then the winner after this deviation must be candidate j because $j \in \hat{S}(E, \sigma_{-1})$. In both cases, candidate i's payoff is $v^i(j) > v^i(j) - d$, which implies that

candidate i has an incentive to deviate. ■

Lemma 18 *Suppose that $CW = \{i\}$ and $GW = \{j\}$. If $i \neq 1$ and $v^M(i) < v^M(j)$, then $\sigma_i(\hat{h}_i(E, \sigma_{-1})) = N$ for any SPE σ.*

Proof of Lemma 18 Suppose, in contrast, that there exists an SPE σ such that $\sigma_i(\hat{h}_i(E, \sigma_{-1})) = E$. By Lemma 17, $C(\hat{S}(E, \sigma_{-1})) = \{i\}$. Because $i \notin GW$, there exists candidate $k \in \{j, M\}$ such that $\sigma_k(\hat{h}_k(E, \sigma_{-1})) = N$. In play (E, σ_{-1}), candidate k's payoff is $v^k(i)$. Note that $v^k(j) - d > v^k(i)$ for any $k \in \{j, M\}$ because of $v^M(j) > v^M(i)$ and Assumptions 1 and 3. Now, we show that candidate k's deviation to $a_k = E$ at history $\hat{h}_k(E, \sigma_{-1})$ yields a higher payoff $v^k(j) - d$, which implies that he has an incentive to deviate. Note that $k \neq 1$ and $k \neq i$ because $\sigma_k(\hat{h}_k(E, \sigma_{-1})) = N$. There are the following two cases to be checked.

1. Suppose that $k = 2$, which implies $i = 3$. Candidate k's deviation induces both candidates 1 and 2 to be entrants. Given history $\hat{h}_i = \{1, 2\}$, because $C(\{1, 2\}) = C(\mathcal{N} \setminus \{i\}) = C(\mathcal{N}) = \{j\}$, candidate i prefers to exit, by which he saves the entry cost. Then candidate k's payoff is $v^k(j) - d$.

2. Suppose that $k = 3$. Because $i \neq 1$ and $\sigma_i(\hat{h}_i(E, \sigma_{-1})) = E$, $\hat{h}_k(E, \sigma_{-1}) = \{1, i\}$. Hence, candidate k's entry makes candidate j become the winner, and then this deviation gives candidate k payoff $v^k(j) - d$. ■

B.1.4 Proof of Theorem 4

Lemmas 1, 2, 3, 4 and 5 are implied by Lemmas 7, 13, 15, 16 and 18, respectively. Then the proof in Appendix A.2 covers the case where $\#(CW) = \#(GW) = 1$ with $CW \neq GW$. In the following, we complete the proof by investigating cases in which (i) $\#(CW) = 1$ and $\#(GW) = 2$ (Proposition 3); and (ii) $\#(CW) = 2$ and $\#(GW) = 1$ (Proposition 4).

Proposition 3 *Suppose that $CW = \{i\}$ and $GW = \{i, j\}$. Then, $C(\hat{S}(\sigma)) = GW$ (resp. CW) for any SPE σ if $v^M(i) < v^M(j)$ (resp. $v^M(i) > v^M(j)$) holds.*

Proof of Proposition 3 By Lemma 8, an SPE must satisfy $i \in \hat{S}(\sigma)$. Because Lemma 7 implies $M \notin C(\hat{S}(\sigma))$, $C(\hat{S}(\sigma))$ is either CW or GW. If $v^M(i) < v^M(j)$, then because Lemma 11 implies $\hat{S}(\sigma) = \mathcal{N}$, we have $C(\hat{S}(\sigma)) = GW$. Suppose on the other hand $v^M(i) > v^M(j)$. Lemma 12 implies $\hat{S}(\sigma) \neq \mathcal{N}$. Furthermore because $j \notin CW$, $C(\{i, j\}) \neq \{i, j\}$. These imply $C(\hat{S}(\sigma)) \neq GW$ and then $C(\hat{S}(\sigma)) = CW$. ■

Proposition 4 *Suppose that $CW = \{i, j\}$ and $GW = \{j\}$. Then, $C(\hat{S}(\sigma)) = CW$ (resp. GW) for any SPE σ if $v^M(i) > v^M(j)$ (resp. $v^M(i) < v^M(j)$).*

Proof of Proposition 4 Lemmas 7 and 8 imply $M \notin C(\hat{S}(\sigma))$ and $j \in \hat{S}(\sigma)$. Note that because $j \in C(S)$ whenever $j \in S$, $C(\hat{S}(\sigma))$ is either CW or GW.

Suppose first that $C(\hat{S}(\sigma)) = CW$ when $v^M(i) < v^M(j)$. Then $\hat{S}(\sigma) = CW = \{i, j\}$ must hold. Note that because $\sigma_M(\hat{h}_M(\sigma)) = N$, M's equilibrium payoff is $\left(v^M(i) + v^M(j)\right)/2$. Now, suppose that M deviates to $a_M = E$ at history $\hat{h}_M(\sigma)$. Because $\sigma_j(\hat{h}_j(\sigma)) = E$ by Lemma 8 and $i \notin GW$, the winner after this deviation is certainly candidate j, and M's payoff from this deviation is $v^M(j) - d$. Because of $v^M(i) < v^M(j)$ and Assumptions 1 and 3, $v^M(j) - d > \left(v^M(i) + v^M(j)\right)/2$, meaning that M prefers this deviation, which is a contradiction.

Suppose next that $C(\hat{S}(\sigma)) = GW$ when $v^M(i) > v^M(j)$. Because $M \notin \hat{S}(\sigma)$ by Lemma 16, $i \in CW \setminus GW$ implies $\hat{S}(\sigma) = \{j\}$, and then candidate i's equilibrium payoff is $v^i(j)$. Now, suppose that candidate i deviates to $a_i = E$ at history $\hat{h}_i(\sigma)$. If candidate i is the follower of M, then the set of the entrants is $\{i, j\}$ because of Lemma 8 and $M \notin \hat{h}_i(\sigma)$. If candidate i is the leader of M, then Lemma 14 implies that M does not enter after observing i's entry. Then, in both cases, the set of the winners after candidate i's deviation is CW and candidate i's payoff is $(v^i(i) + v^i(j))/2 - d$, which is greater than

the equilibrium payoff $v^i(j)$ because of Assumptions 1 and 3. Hence, candidate i has an incentive to deviate, which is a contradiction. ■

B.1.5 Proof of Theorem 5

(Necessity) Note that by Lemmas 15 and 16, condition (b) should hold if strategic candidacy occurs in equilibrium. Hence, it remains to show condition (a). Suppose, in contrast, that there exists an SPE σ such that $\hat{S}(\sigma) \neq C(\hat{S}(\sigma))$ when M is a follower of candidate i. Given $v^M(j) > v^M(i)$, Theorem 4 implies $C(\hat{S}(\sigma)) = GW = \{j\}$. Then, because $\sigma_M(\hat{h}_M(\sigma)) = E$, M's equilibrium payoff is $v^M(j) - d$. Now, suppose that M deviates to $a_M = N$ at history $\hat{h}_M(\sigma)$. Because M is a follower of candidate i and $i \notin \hat{h}_M(\sigma)$ by Lemma 15, candidate j is still the winner even after the deviation. Then, M's payoff from this deviation is $v^M(j) > v^M(j) - d$, implying that M has an incentive to deviate.

(Sufficiency) Suppose that conditions (a) and (b) hold. Because of Theorem 4, $C(\hat{S}(\sigma)) = \{j\}$ holds for any SPE σ. Furthermore, because $i \notin \hat{S}(\sigma)$, by Lemma 15, for any SPE σ, $\hat{S}(\sigma)$ is either $\{j\}$ or $\{M, j\}$. Suppose now that there exists an SPE σ such that $\hat{S}(\sigma) = \{j\}$. Because $\sigma_i(\hat{h}_i(\sigma)) = N$, candidate i's equilibrium payoff is $v^i(j)$. Now, suppose that candidate i deviates to $a_i = E$ at history $\hat{h}_i(\sigma)$. Because M is a leader of candidate i, $M \notin \hat{h}_i(\sigma)$, and $i \in CW$, the set of the winners by this deviation is either $\{i\}$ or $\{i, j\}$ and then candidate i's payoff is either $v^i(i) - d$ or $(v^i(i) + v^i(j))/2 - d$. In each case, the deviation payoff is strictly greater than the equilibrium payoff $v^i(j)$ by Assumptions 1 and 3, implying that candidate i has an incentive to deviate. Therefore σ must satisfy $\hat{S}(\sigma) = \{M, j\}$. ■

B.1.6 Proof of Theorem 6

The necessity is straightforward from Lemmas 15 and 16. To show the sufficiency, suppose that $v^M(i) < v^M(j)$. By Proposition 3, $C(\hat{S}(\sigma)) = GW$, implying that $S(\hat{\sigma}) = \mathcal{N}$ for any SPE σ because $j \notin CW$. Thus, $\hat{S}(\sigma) \neq C(\hat{S}(\sigma))$ for any SPE σ. ■

B.1.7 Proof of Theorem 7

(Necessity) Suppose, in contrast, that M threatens candidate i in SPE σ when $v^M(i) > v^M(j)$. Theorem 1 and Proposition 4 imply $C(\hat{S}(\sigma)) = CW$, and Theorem 5 implies $M \notin \hat{S}(\sigma)$. Then $\hat{S}(\sigma) = \{i\}$ when $\#(CW) = 1$ and $\hat{S}(\sigma) = \{i, j\}$ when $\#(CW) = 2$. In both cases, because candidate i enters and M does not enter on the equilibrium path, M does not threaten candidate i. Hence, $v^M(i) < v^M(j)$ must hold.

(Sufficiency) Suppose that $v^M(i) < v^M(j)$. Then, Theorem 1 and Proposition 4 imply $C(\hat{S}(\sigma)) = GW = \{j\}$. Because M makes a decision after candidate i, Theorem 5 implies $M \notin \hat{S}(\sigma)$ for any SPE σ. Because $C(\{i, j\}) \neq GW$, $\hat{S}(\sigma) = \{j\}$ must hold, which means that both M and candidate i exit on the equilibrium path. Hence, it remains to show that at history $\hat{h}_i(\sigma)$, if candidate i deviates to enter, then M also chooses to enter. Suppose, in contrast, that at history $\hat{h}_i(\sigma)$, if candidate i deviates to enter, then M chooses to exit. Then, because the set of the entrants is either $\{i\}$ or $\{i, j\}$ and $i \in CW$, the set of the winner is either $\{i\}$ or $\{i, j\}$. As a result, candidate i's payoff is either $v^i(i) - d$ or $(v^i(i) + v^i(j))/2 - d$ and both of them are strictly greater than the equilibrium payoff $v^i(j)$ by Assumptions 1 and 3. This contradicts that σ is an SPE. ■

B.2 Endogenous Decision Order

In this subsection, we analyze a model where the decision order is endogenously determined in equilibrium, and insist that the kingmaker property of the minor still hold.

B.2.1 The Temporal Game

The modified model is called the *temporal game* following Osborne (1993).[19] There is an infinite sequence of periods $t = 1, 2, \ldots$. In each period, the candidates who have not yet finalized the entry decision simultaneously decide upon an action. Let $a_i^t \in A \equiv \{E, N, P\}$

[19] In contrast with Osborne (1993), the candidates cannot commit any policy announcement here.

be the action of candidate $i \in \mathcal{N}$ at period t, where $a_i^t = E$ (resp. N) means that candidate i commits to enter (resp. not enter) at period t, and $a_i^t = P$ means that candidate i postpones his decision at period t. We say that candidate i is *active* at period t if $a_i^{t'} = P$ for any $t' < t$; otherwise, he is called *inactive*. Let $a_i^0 = P$ for each i. The election takes place immediately after all the candidates become inactive. As in the baseline model, each candidate's payoff is determined based on the electoral outcome and the entry decision (without time discounting).

Candidate i's pure strategy σ_i is a mapping from the set of all possible histories to A. As before, let $\hat{S}(\sigma)$ be the set of entrants in the election. We assume that if a strategy profile $\sigma \equiv (\sigma_1, \sigma_2, \sigma_3)$ specifies $a_i^t = P$ for every t and some i on the equilibrium path, then $i \notin \hat{S}(\sigma)$, that is, candidate i stays out of the election. Hereafter, we focus on SPE.

For simplicity, we focus on cases with a unique Condorcet winner and a unique grand winner. We modify the definition of the kingmaker as follows.

Definition 5 *Candidate i is a kingmaker under voting procedure $C(\cdot)$ if for any preference satisfying Assumption 1, there exists an equilibrium σ under $C(\cdot)$ in which $v^i(j) > v^i(k)$ for $j, k \in \mathcal{N} \backslash \{i\}$ and $j \neq k$ implies $j = C(\hat{S}(\sigma))$.*

The modified definition requires the *existence* of equilibria such that the kingmaker's preferred rival wins for any preference of candidates.[20]

B.2.2 Analysis

First, we can observe the following difference from the baseline model.

Proposition 5 *There always exists SPE σ where CW is the unopposed winner.*

Proof of Proposition 5 Without loss of generality, let $CW = 1$. It is sufficient to show that $a^1 \equiv (a_1^1, a_2^1, a_3^1) = (E, N, N)$ is supported in equilibrium. It is obvious that

[20] The baseline model in the body of the paper, we have shown a more robust result in that the kingmaker's preferred rival wins for *any* equilibrium.

CW has no incentive to deviate to any strategy that prevents his entry because he wins. Moreover, candidate $i(=2,3)$ also has no incentive to deviate to any strategy that induces himself to enter because it does not change the electoral outcome. ■

In contrast with the fixed order environment, there always exists an equilibrium in which CW is the unopposed winner whatever the minor's preference is. Because the active candidates simultaneously decide in each period, a Nash equilibrium outcome of the simultaneous entry games can be supported as an SPE outcome. However, in contrast with the simultaneous entry games, there also exists a pure strategy equilibrium in which GW wins given that CW does not coincide with GW and M prefers GW.

Proposition 6 *Suppose that $CW \neq GW$ and $v^M(GW) > v^M(CW)$. Then there exists SPE σ such that $\hat{S}(\sigma) = \{GW, M\}$.*

Proof of Proposition 6 Given history in period t denoted by h_t, let $\phi(h_t) \equiv (\phi_E(h_t), \phi_N(h_t))$, where $\phi_E(h_t)$ (resp. $\phi_N(h_t)$) be the set of candidates who commit to enter (resp. not to enter). Consider the following strategies: $(\sigma_{CW}(h_1), \sigma_{GW}(h_1), \sigma_M(h_1)) = (P, P, E)$ and for $t \geq 2$,

$$\sigma_{CW}(h_t) = \begin{cases} N & \text{if } M \in \phi_E(h_t) \text{ and } GW \notin \phi_N(h_t), \\ E & \text{otherwise,} \end{cases}$$

$$\sigma_{GW}(h_t) = \begin{cases} E & \text{if } M \in \phi_E(h_t) \text{ or } CW \in \phi_N(h_t), \\ N & \text{otherwise,} \end{cases}$$

$$\sigma_M(h_t) = \begin{cases} E & \text{if } \phi_E(h_t) = \{CW, GW\} \text{ or } \phi_N(h_t) = \{CW, GW\} \\ N & \text{otherwise.} \end{cases}$$

We check that given h_t for $t \geq 2$, the strategy profile constitutes a Nash equilibrium. Suppose first that two of three candidates are inactive. Then since this is the individual decision problem for the active candidate, it is easy to check that the above strategy

constitutes a Nash equilibrium. Second, suppose that there is exactly one candidate who is inactive.

1. Suppose $\phi_E(h_t) = \{CW\}$. Then the above strategy specifies $\sigma_{GW}(h_t) = \sigma_M(h_t) = N$. Given that candidate $j(= GW, M)$ chooses N, CW is the winner in the election regardless of the action by candidate $i(\in \{GW, M\} \setminus \{j\})$. Then candidate i prefers to choose N for saving the entry cost. Then the strategy profile constitutes a Nash equilibrium.

2. Suppose $\phi_E(h_t) = \{GW\}$. Then the above strategy specifies $\sigma_{CW}(h_t) = E$ and $\sigma_M(h_t) = N$. Given that M chooses N, CW prefers to chooses E because he wins in a pairwise vote against GW. Given that CW chooses E, M prefers to exit because CW is the winner regardless of M's action. Then the strategy profile constitutes a Nash equilibrium.

3. Suppose $\phi_N(h_t) = \{j\}$ for $j = CW, GW$. Let $i \in \{CW, GW\} \setminus \{j\}$. Then the above strategy specifies $\sigma_i(h_t) = E$ and $\sigma_M(h_t) = N$. Given that M chooses N, candidate i prefers to chooses E because he becomes the unopposed winner. Given that candidate i chooses E, M prefers to exit because candidate i is the winner regardless of M's action. Then the strategy profile constitutes a Nash equilibrium.

4. Suppose $\phi_E(h_t) = \{M\}$. Then the above strategy specifies $\sigma_{CW}(h_t) = N$ and $\sigma_{GW}(h_t) = E$. Given that GW chooses E, because CW cannot win in the three-entrant vote, the winner is GW regardless of CW's action. Then CW prefers to exit to save the entry cost. Given that CW chooses N, GW prefers to enter because he beats M in the vote. Then the strategy profile constitutes a Nash equilibrium.

5. Suppose $\phi_N(h_t) = \{M\}$. Then the above strategy specifies $\sigma_{CW}(h_t) = E$ and $\sigma_{GW}(h_t) = N$. Given that CW chooses E, because GW cannot win CW in the pairwise vote, the winner is CW regardless of GW's action. Then GW prefers

to exit to save the entry cost. Given that GW chooses N, CW prefers to enter because he is the unopposed winner. Then the strategy profile constitutes a Nash equilibrium.

Third, suppose that all the candidates are active. Then the above strategy specifies $\sigma_{CW}(h_t) = E$ and $\sigma_{GW}(h_t) = \sigma_M(h_t) = N$. The proof of Proposition 5 implies that this constitutes a Nash equilibrium.

Now consider decisions in period 1. Given $(a^1_{CW}, a^1_{GW}, a^1_M) = (P, P, E)$, because CW and GW choose N and E, respectively, in period 2, the payoffs of CW, GW, and M from following a^1 are $v^{CW}(GW)$, $v^{GW}(GW) - d$, and $v^M(GW) - d$, respectively. We will check that each candidate has no incentive to deviate.

1. Suppose that CW deviates to either E or N. Then the strategy specifies that GW chooses E in period 2. Because M also enters, GW wins regardless of CW's action and then CW's payoff is $v^{CW}(GW) - d$ (when he deviated to E) or $v^{CW}(GW)$ (when he deviated to N), which is not grater than his equilibrium payoff.

2. Suppose that GW deviates to E. Then the strategy specifies that CW chooses N in period 2. Then GW wins and then his payoff is $v^{GW}(GW) - d$, which is not grater than his equilibrium payoff.

3. Suppose that GW deviates to N. Then the strategy specifies that CW chooses E in period 2. Then CW wins, which is less preferred to the equilibrium outcome by GW.

4. Suppose that M deviates to either N or P. Then the strategy specifies that CW chooses E and GW chooses N in period 2. Then CW wins and then M's payoff is $v^M(CW)$, which is not grater than his equilibrium payoff $v^M(GW) - d$ by Assumption 1. ■

As in the baseline model, if M prefers GW to CW, then there exists an equilibrium where M attempts strategic candidacy to crowd out CW. Consequently, M can ensure that GW wins the election. Therefore, combining Propositions 5 and 6 implies that M is a kingmaker in the sense of Definition 5.

Theorem 8 *In the temporal game, as long as $CW \neq GW$, M is a kingmaker under any voting procedure.*