Manipulated News Model: Electoral Competition and Mass Media *

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Abstract

This paper investigates the relationship between *media manipulation*, the strategic suppression of relevant information by mass media, and *self-mediatization*, the strategic exploitation of media coverage by candidates, in elections. In the model, a voter cannot directly observe the policies proposed by two office-motivated candidates. The voter learns this information through media reports before voting takes place, while the media outlet suppresses some of this information. Because the voter's decision-making could be incorrect (*direct distortion*), the candidates have an incentive to win the election by influencing the media coverage through policy settings (*indirect distortion*). As a result, policy convergence to the voter's ideal policy occurs if and only if the media bias is sufficiently small. We then characterize the set of equilibria in terms of the degree of distortion. The results suggest that if either strategic media manipulation or competition among the candidates is omitted, then the distortion is nonnegligibly misspecified.

Journal of Economic Literature Classification Numbers: C72, D72, D82.

Key Words: Downsian voting model; media manipulation; self-mediatization; persuasion games; direct/indirect distortion; equilibrium set; misspecification

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1 Introduction

The mass media has a substantial influence on political outcomes. In modern elections, the interactions between candidates and voters are indirect in the sense that the mass media plays an informative role between them and, thus, provides essential information for their decision-making. For instance, most voters use the news as an information source for voting instead of directly acquiring relevant information. Likewise, candidates decide the content of their electoral campaigns after taking account of polls. That is, the mass media can influence electoral outcomes by acting as an intermediary in the transmission of information between candidates and voters.

Because of their informational advantage, media outlets may have an incentive to manipulate the content of released news to influence electoral outcomes. For example, in the 1999 parliament election in Russia, Unity, a pro-government party, and Fatherland-All Russia (hereafter, OVR), the most popular opposition party, competed for the support of centrists.[1] According to the European Institute for the Media (2000), most TV programs focused on a particular party with biased coverage, and comparisons of party policies were little reported. In fact, ORT, a state-controlled outlet that was the broadcast flagship of Russia, devoted 28% and 15% of the news time to Unity and OVR with positive and negative spins, respectively. On the contrary, NTV, a major independent commercial outlet whose political position was anti-government, devoted 5% and 33% of the news time to Unity and OVR with negative and positive spins, respectively.[2] A positive or negative spin can be regarded as the suppression of election-relevant information because it only highlights positive or negative aspects instead of providing unbiased reporting, which is just one of several common forms of strategic manipulation by the mass media.[3]

Similarly, candidates also strategically decide their behaviors by internalizing how they are reported by the mass media. In the 2016 Republican Party presidential primaries, for instance, Donald Trump made several politically incorrect statements, which could be understood as his campaign strategy to exploit media coverage. These sensational statements caused mainstream media outlets to increase their coverage of Trump because they were nonstandard with high news value, which could have positively affected his success (e.g., Simon 2016).[4] Furthermore, Trump

[1]The detailed background is summarized in Enikolopov et al. (2011).

[2]As reported by the European Institute for the Media (2000), the head of NTV admitted that its programs had positive bias toward OVR to counter the negative coverage by ORT.

[3]Bagdikian (1997) argues that "[e]very basic step in the journalistic process involves a value-laden decision: Which of the infinite number of events in the environment will be assigned for coverage and which ignored? Which of the infinite observations confronting the reporter will be noted? Which of the facts noted will be included in the story? ...None of these is a truly objective decision."

[4]The literature on political science points out that rises in a candidate's media coverage, even negative coverage, positively affect his or her success in the campaign. See Burden (2002) and Shen (2008).

seemed to adopt such offensive rhetoric with strategic intent. As pointed out by Freeman (2016), his statements were similar to those by Pat Buchanan, who defeated Trump in the 2000 Reform Party presidential preliminaries, when Trump labeled Buchanan's statements "beyond far right." The difference in Trump's stance could suggest that he strategically adopted the rhetoric to win the election by exploiting media coverage.

This paper investigates the interaction between *media manipulation*, namely the strategic suppression of relevant information by the mass media, and *self-mediatization*, namely the strategic exploitation of media coverage by candidates.[5] These two phenomena interact with each other in elections, which could distort electoral outcomes. Imagine, for example, a major media outlet that strongly endorses the reduction of military expenditure. As in the Russian example, it might be natural for such an outlet to put positive spins on candidates who agree with the policy of reducing military expenditure, which may prevent voters from forming correct perceptions of the candidates. Furthermore, if candidates were aware of the stance of the media outlet, they would then display affirmative attitudes towards the reduction of military expenditure to increase their media coverage, as in the Trump example. Hence, candidates' policies tend to be biased toward the outlet's preferences. In other words, the outcome could be affected not only by distorting voters' observation, but also changing the alternatives available to voters. Therefore, clarifying the "severity" of the distortion of the electoral outcomes caused by this interaction is important, in order to understand the effect of mass media.

While media manipulation and self-mediatization in the above sense are commonplace in the literature, the overall extent of their interaction remains unknown. Most existing models omit the competition among candidates or the strategic aspect of the mass media. Although these are useful simplifications, they come at a cost. As in the above examples, media manipulation tends to be strategic, and candidates strategically exploit it to beat the competition; this interaction is fed back to media manipulation through changing voting behaviors, and so on. Hence, to correctly evaluate the severity of the distortion, we require a model in which the behaviors of candidates, media outlets, and voters are all determined endogenously. Otherwise, we may over- or underestimate the severity. Thus, this paper has two objectives. First, we develop a tractable model of electoral competition, including both strategic media manipulation and competition among candidates. Second, we clarify how media manipulation and self-mediatization interact with each other, and evaluate the overall severity of the distortion, using the proposed model.

[5] According to Esser (2013), in the context of media communication, the notion of self-mediatization describes the phenomenon in which "[p]oliticians have internalized the media's attention rules, production routines, and selection criteria, and try to exploit this knowledge for attending political goals." See also Meyer (2002).

We consider the following Downsian voting model, including media outlets. There are two office-motivated candidates, a single media outlet, and a single voter, all of whom are rational.[6] Unlike standard models, we assume that the voter cannot directly observe the policies proposed by the two candidates. Instead, the voter learns this information through reports from the media outlet. In other words, we consider the following two-stage game. In the first stage, the candidates simultaneously propose policies that only the media outlet observes. In the second stage, the media outlet, whose preference differs from that of the voter, decides on the release of the information about the proposed policies, after which the voter chooses one of the candidates.

The results are as follows. First, we demonstrate that distortion occurs in the equilibrium outcomes compared with the scenario in which no manipulation occurs because of the following mechanism. Again, consider the situation where two candidates compete in an election whose main issue is the reduction of military expenditure, and the (mainstream) media outlet has a bias toward drastic reduction. Now, suppose that the outlet puts positive and negative spins on candidates 1 and 2, respectively (media manipulation). When the (representative) voter observes such biased news, he attempts to infer why it is so biased. However, this inference might be imperfect; from the voter's perspective, there are two possible reasons for the biased news: (i) candidate 1 is better for both the voter and the outlet, or (ii) candidate 2 is better for the voter but his positive aspects are suppressed because the outlet prefers candidate 1 and wants to make it appear that candidate 1 is also better for the voter. Even though the voter is fully rational, he cannot definitively identify the reason. Because of this indeterminacy, the voter might choose the ex post unfavored candidate with some positive probability, which represents the distortion on the voter's behavior (*direct distortion*).

Because of the voter's ex post incorrect decision-making, proposing the voter's ideal policy becomes less attractive to the candidates, who then have an incentive to win the election by influencing the media outlet's behavior through policy settings (self-mediatization). Suppose that the voter believes reason (i), and he then chooses candidate 1 as a response to the biased news. That is, candidate 1 has an advantage because of the biased news. In this scenario, candidate 1 is incentivized to agree with the drastic reduction in military expenditure to induce the biased news, thereby maintaining his advantage. On the contrary, candidate 2 would also agree with the drastic reduction in military expenditure to avoid being disadvantaged by the biased news. As a result, policies that are not ideal for the voter could occur in equilibrium, which represents a distortion of the candidates' behavior (*indirect distortion*). In fact, because the incentive structure is similar to that of matching pennies games in the sense that one candidate attempts to induce the biased news

[6] Throughout the paper, we treat candidates and voters as males and outlets as females.

while the other attempts to avoid it, mixed-strategy equilibria exist in the first stage, which induces policy divergence on the equilibrium path. Furthermore, because of the distortion channels, there exist multiple equilibria, but convergence to the voter's ideal policy cannot be supported in an equilibrium when the bias of the media outlet is significant, in contrast with the standard model.

Second, we focus on the equilibria constructed by undominated strategies in some sense and characterize the equilibrium set in terms of the degree of distortion measured by the voter's ex ante expected utility. Here, we specify the least and most distorted scenarios, and show that any value between these bounds can be supported as the equilibrium distortion. Then, the comparative statics show that as the outlet becomes more biased, the distortion becomes more severe in that both the lower and the upper bounds of the distortion are nondecreasing in the level of media bias, and the equilibrium outcome becomes more dispersed. Likewise, the equilibrium outcome is more dispersed as the candidates behave more opportunistically.

Finally, we compare the baseline model with the reduced form models in which either competition among the candidates or strategic media manipulation is omitted, and then discuss the extent to which such simplifications misspecify the severity of the distortion. First, in the model in which the behavior of either one of the candidates is exogenously fixed, the outcome is sensitive to the setup. Furthermore, because we can ignore the incentive compatibility of the nonstrategic candidates, the first-best outcome is approximately attainable. That is, the distortion could be underestimated in this simplification. Second, we consider the model in which media manipulation is represented by an exogenously fixed nondegenerate distribution. Because the information is disclosed with positive probability, the indirect distortion is eliminated. However, the direct distortion can be mitigated or exaggerated depending on the media bias of the baseline model. That is, if the bias is small (resp. large), then the reduced form model overestimates (resp. underestimates) the distortion. Thus, we conclude that these simplifications, frequently used in the literature, generate nonnegligible misspecification.

The remainder of the paper is organized as follows. In the following subsection, we briefly review the related literature. Section 2 defines and discusses the formal model. Section 3 analyzes a benchmark model without media manipulation, and Sections 4 and 5 analyze a model with media manipulation. We clarify the distortion mechanism in Section 4, and characterize the set of equilibria in Section 5. Lastly, we discuss the misspecification of the reduced models in Section 6, and conclude in Section 7. The proofs are in Appendix A.[7]

[7]The Supplementary Appendix, referred to as Appendix B, contains omitted proofs, discussions, and extensions.

1.1 Related literature

This paper is mostly related to the literature on the political economics of mass media.[8] We can divide the literature on election models including the mass media into two strands depending on the role of the mass media. In the first strand, media outlets are modeled as "outside observers" that provide additional election-relevant information rather than distorting the information transmission between candidates and voters. In other words, voters update their beliefs about payoff-relevant uncertainty by observing both candidate behavior and the reports provided by media outlets. For example, Chan and Suen (2008, 2009) and Gul and Pesendorfer (2012) consider a two-candidate election model in which media outlets endorse one of the candidates by sending cheap-talk messages, and investigate the relationship between political polarization and media competition.[9] Elsewhere, Ashworth and Shotts (2010) and Warren (2012) examine a retrospective voting model in which the incumbent politician has reputational concerns and the media outlets, again, provide cheap-talk endorsements.[10] Because the cheap-talk endorsements could transmit credible information even if the outlets are biased, these studies show that media outlets improve voter welfare.[11] However, Chakraborty and Ghosh (2016) obtain the opposite implication by considering a model in which outlets send cheap-talk endorsements about candidates' character that are unobservable to voters. Because the candidates distort their platforms toward the outlet's preferred direction to obtain the endorsement, they conclude that the value of mass media can be negative.

In the second strand of the literature, media outlets are modeled as "intermediaries" in the information transmission process. That is, media manipulation could distort voters' observations. This strand is further divided into the subgroups of the strategic and nonstrategic mass media. In the case of the strategic mass media, on the one hand, Bernhardt et al. (2008) and Perego and Yuksel (2018) consider profit-motivated outlets to study media market competition. In Bernhardt et al. (2008), because of media competition, outlets suppress candidates' negative information to cater for partisan voters who dislike negative news about their preferred candidates, which induces the voters' incorrect decision-making. Perego and Yuksel (2018) obtain a similar conclusion with-

[8]According to a survey by Prat and Strömberg (2013), key theoretical areas of research include (i) media capture by the government (Besley and Prat 2006), (ii) how the mass media affects government public policy (Strömberg 2004), and (iii) how media bias is generated (Mullainathan and Shleifer 2005; Baron 2006; Gentzkow and Shapiro 2006). An overview of each area is provided, for instance, by Prat (2016), Strömberg (2016), and Gentzkow et al. (2016), respectively.

[9]Bandyopadhyay et al. (2016) demonstrate that political polarization could be exaggerated, even though the outlet is a monopolist and does not have a partisan preference.

[10]While the outlets' information is exogenous in Ashworth and Shotts (2010), it is endogenous in Warren (2012).

[11]Andina-Díaz (2009a, b) investigates the relationship between media competition and information acquisition by media outlets under the assumption that media outlets reveal everything what they know. In this environment, the value of mass media is positive because such outlets provide additional information, but its degree is dependent on their motivation and voters' purchasing habits.

out making behavioral assumptions on the voters' preferences. They consider that media outlets provide noisy information about candidates' valence and ideology, concluding that the competition makes the voters worse off because it causes each outlet to report only the ideological aspect. On the other hand, Duggan and Martinelli (2011) consider policy-motivated outlets, and develop a retrospective voting model in which these outlets reduce two-dimensional policy information into a one-dimensional "story" through media manipulation. That is, they assume that full information revelation on both dimensions is impossible. Thus, compared with balanced outlets that report both policy dimensions with equal weight, a biased outlet can improve social welfare by revealing full information about either of the dimensions.

For the nonstrategic mass media, Adachi and Hizen (2014) analyze a retrospective voting model in which media outlets systematically add noise to voters' observations. They show that media bias, even anti-incumbent bias, never improves social welfare because, independent of its direction, the bias erodes the credibility of bad news, which prompts corruption by the incumbent. Pan (2014) develops a two-candidate Wittman model with a noise structure similar to that of Adachi and Hizen (2014), and clarifies the mechanism for political polarization. As in Adachi and Hizen (2014), the noise into the reports causes candidates to propose ideological policies more frequently, which could reduce the voter's welfare.

Two remarks are relevant with regard to how this body of work is related to the current analysis. First, we can neatly classify our analysis as belonging to the second strand of inquiry.[12] However, in contrast to other studies, our model includes both competition between candidates and strategic media manipulation. Because of the difference in the scope of their studies, Bernhardt et al. (2008), Duggan and Martinelli (2011), and Perego and Yuksel (2018) exogenously fix the proposed policies; that is, they focus on media manipulation by omitting the aspect of self-mediatization in competition. Likewise, Adachi and Hizen (2014) and Pan (2014) focus on self-mediatization by omitting the strategic aspect of media manipulation. Thus, the literature does not fully answer the question of how and to what extent the equilibrium outcomes are distorted because of the interaction of these two aspects. Therefore, we present a model that assumes that the candidates, media outlets, and voters are all fully rational.

Second, this paper is a complement of Chakraborty and Ghosh (2016), which also investigate the relationship between media manipulation and self-mediatization. However, the role of the mass media in their study is different from that in ours. On the one hand, Chakraborty and Ghosh (2016) include media outlets as outside observers. This setup is associated with the situation in

[12] This paper also demonstrates policy divergence by introducing voter uncertainty, as in Kartik and McAfee (2007) and Kikuchi (2014).

which media outlets reveal their stance as "endorsements" in addition to "news reports," as in the US.[13] That is, the voter has two channels through which to obtain election-relevant information, and then the distortions in policies and information are separable in the sense that each distortion affects the voter's welfare through a different communication channel. On the other hand, we model media outlets as intermediaries: each outlet does not provide formal endorsements, but its stance is informally reflected in the contents of news reports, as in the Russian example mentioned above. Because the voter has only one communication channel, these distortions are not separable. As a result, in addition to the difference in applications, the structures of the equilibria are also different.[14]

To describe the suppression of information by media outlets, we adopt a persuasion (or disclosure) game framework from the strategic communication literature.[15] Persuasion games are sender–receiver games with hard private information, as first formalized by Milgrom (1981), for which there is now a large volume of the literature. See, for example, Milgrom and Roberts (1986), Seidmann and Winter (1997), Giovannoni and Seidmann (2007), and Hagenbach et al. (2014). In contrast to cheap-talk games à la Crawford and Sobel (1982), the sender is unable to misrepresent information, but can conceal unfavorable information. Based on Miura (2014, 2016, 2018), we analyze a hierarchical persuasion game in which the sender's private information is affected by the strategies of others.[16]

2 The Model

We define the baseline model in Section 2.1, and discuss its plausibility in Section 2.2.

2.1 Setup

There are four players in our model: candidates 1 and 2, a single media outlet, and a single voter. The players play the following two-stage game. In the first stage, called the *policy-setting stage*, each

[13] According to The New York Times (2016), the news department producing news reports and the editorial board writing endorsements are completely separated. Furthermore, the news department tries to keep the news balanced.

[14] First, the policy convergence to the voter's ideal policy never occurs in Chakraborty and Ghosh (2016, Proposition 1), but it occurs when the bias is sufficiently small in our setup. Second, in Chakraborty and Ghosh (2016), pure-strategy equilibria never exist when the bias is sufficiently large enough, but they exist in our paper. Finally, while only symmetric equilibria exist in Chakraborty and Ghosh (2016), asymmetric equilibria also exist in our setup.

[15] Bernhardt et al. (2008) and Anderson and McLaren (2012) also adopt a simple persuasion game to describe suppression by media outlets. In contrast to Bernhardt et al. (2008) and this paper, the main scope of Anderson and McLaren (2012) is media mergers.

[16] Other types of hierarchical communication are also studied in the literature. See, for example, Ivanov (2010), Li (2010), and Ambrus et al. (2013). Recently, Ben-Porath et al. (2018) also consider a hierarchical persuasion game in which the sender's private information is affected by the other players, called *agents*. A difference from our model is that they assume only one agent, and competition among the agents is beyond the scope of their paper.

candidate simultaneously proposes a policy, and only the outlet observes these proposed policies. In the second stage, called the *news-reporting stage*, the outlet sends a message about the proposed policies to the voter. After observing the message, the voter casts a ballot for one of the candidates. The winning candidate then implements his proposed policy.

Let $X \equiv [-\bar{x}, \bar{x}] \subset \mathbb{R}$ be the set of available policies for the candidates with $\bar{x} > 0$. Let $x_i \in X$ be the policy proposed by candidate $i \in \{1, 2\}$, and $z \equiv (x_1, x_2) \in Z \equiv X^2 \subset \mathbb{R}^2$ describe a policy pair proposed by the candidates. We assume that the media outlet, but not the voter, correctly observes policy pair z. Hence, the information about policy pair z is the media outlet's private information in the news-reporting stage. Furthermore, we assume that the outlet cannot fabricate this information. To represent this assumption, the message space, given policy pair z, is defined by $M(z) \equiv \{m \in 2^Z | z \in m\}$.[17] That is, the available messages under policy pair z are subsets of policy pair space Z containing the truth z.[18] Let $m \in M \equiv \bigcup_{z \in Z} M(z)$ be a message from the outlet. Let $y \in Y \equiv \{y_1, y_2\}$ be the action of the voter, where y_i denotes that the voter casts a ballot for candidate i.

We assume two types of candidates: an *opportunistic candidate* and an *ideological candidate*. The opportunistic candidate is the standard office-motivated strategic type. By contrast, the ideological candidate is a nonstrategic type of candidate who always proposes a particular policy. We assume that if candidate 1 (resp. 2) is of the ideological type, then he always proposes policy $r \in (0, \bar{x})$ (resp. $l \in (-\bar{x}, 0)$), and $|r| < |l|$. That is, we assume asymmetry between the candidates. Let $\Theta \equiv \{O, I\}$ be the candidates' type space, while O (resp. I) represents the opportunistic (resp. ideological) type. We assume that candidate i's type $\theta_i \in \Theta$ is candidate i's private information, and that θ_1 and θ_2 are determined independently. Let $p \in (0, 1)$ be the probability that each candidate is an opportunistic type, and assume that this is common knowledge.[19]

We define the players' preferences as follows. Define opportunistic candidate i's von Neumann–

[17]Note that for any subset $P \subseteq Z$, message $m = P$ has the property that $M^{-1}(P) = P$, where $M^{-1}(P)$ represents the set of policy pairs under which message $m = P$ is available. That is, the information about a policy pair is *fully certifiable* in the sense of persuasion games.

[18]We interpret nonsingleton messages as media manipulation by suppressing election-relevant information. Consider, for example, the scenario in which x represents the amount of military expenditure. The singleton message states the exact amount of military expenditure. However, nonsingleton messages simply state that, for instance, "military expenditure is increased" without mentioning the exact amount. That is, the latter scenario can be interpreted as the suppression of the exact amount of military expenditure.

[19]We can obtain qualitatively the same results even if the probability of being the opportunistic type differs among the candidates. The detail is available from the author upon request.

Morgenstern utility function $u_i : Y \to \mathbb{R}$ by:

$$u_i(y) \equiv \begin{cases} 1 & \text{if } y = y_i, \\ 0 & \text{Otherwise.} \end{cases} \tag{1}$$

We assume that the voter and outlet have single-peaked preferences over the implemented policies. Define the voter's von Neumann–Morgenstern utility function $v : Z \times Y \to \mathbb{R}$ by:

$$v(z, y) \equiv \begin{cases} -|x_1| & \text{if } y = y_1, \\ -|x_2| & \text{if } y = y_2. \end{cases} \tag{2}$$

Similarly, define the outlet's von Neumann–Morgenstern utility function $w : Z \times Y \to \mathbb{R}$ by:

$$w(z, y) \equiv \begin{cases} -|x_1 - b| & \text{if } y = y_1, \\ -|x_2 - b| & \text{if } y = y_2. \end{cases} \tag{3}$$

The voter's ideal policy is 0, whereas that of the outlet is $b > 0$. Hence, parameter b represents the difference between the preferences of the voter and of the outlet. We refer to this parameter throughout the paper as the *media bias*. We assume that the actual value of b is common knowledge.

We formalize the timing of the game as follows. In the policy-setting stage, nature chooses candidate i's type $\theta_i \in \Theta$ according to the prior distribution p, and only candidate i correctly learns his own type θ_i. Then, given θ_i, each candidate simultaneously proposes a policy pair $x_i \in X$. Only the outlet correctly observes policy pair $z = (x_1, x_2) \in Z$. In the news-reporting stage, given the observed pair z, the outlet sends a message $m \in M(z)$. After observing the message, the voter undertakes an action $y \in Y$. The policy announced by the winning candidate is then implemented.

We define the players' strategies and the voter's belief as follows. Opportunistic candidate i's strategy is represented by $\alpha_i \in \Delta(X)^*$, where $\Delta(X)^*$ is the set of finite-support probability distributions over the policy space.[20] Let $\alpha_i(x_i)$ represent the probability that candidate i proposes policy x_i. The outlet's strategy $\beta : Z \to \Delta(M)^*$ is a function from an observed policy pair to a finite-support probability distribution over the entire message space.[21] The voter's strategy $\gamma : M \to \Delta(Y)$ is a function from an observed message to a probability distribution over the voter's action set Y. The voter's strategy is represented by $\gamma(m) = (\gamma_1(m), 1 - \gamma_1(m))$, where $\gamma_1(m)$ represents the probability that the voter chooses candidate 1 when he observes message m.

[20] If distributional strategies are allowed, we face serious multiplicity of equilibria. To avoid this problem, we exclude distributional strategies by the candidates.

[21] In contrast to the restriction for the candidates, this restriction is only for technical convenience.

With some abuse of notation, the pure strategies of the players are simply represented by $\alpha_i = x_i$, $\beta(z) = m$, and $\gamma(m) = y$, respectively. Let $\mathcal{P} : M \to \Delta(Z)$ represent the voter's posterior belief, which is a function from an observed message to a probability distribution over the set of proposed policy pairs Z.

We use the perfect Bayesian equilibrium (hereafter, PBE) as a solution concept. Because messages must contain the true policy pair, we add the following requirement as a restriction on off-the-equilibrium-path beliefs. Let $S(f(\cdot))$ be the support of probability distribution $f(\cdot)$.

Requirement 1 *For any message $m \in M$, $S(\mathcal{P}(\cdot|m)) \subseteq m$ holds.*

Definition 1 ***PBE***

A quintuple $(\alpha_1^*, \alpha_2^*, \beta^*, \gamma^*; \mathcal{P}^*)$ is a PBE if it satisfies the following conditions:

(i) for any $i, j \in \{1,2\}$ with $i \neq j$ and any $x_i \in S(\alpha_i^*)$:

$$x_i \in \arg\max_{x_i' \in X} \sum_{x_j \in X} \sum_{m \in M} u_i(y_i) \Pr(y_i|\gamma^*(m)) \Pr(m|\beta^*(x_i', x_j)) \Pr(x_j|\alpha_j^*); \tag{4}$$

(ii) for any $z \in Z$ and any $m \in S(\beta^*(z))$:

$$m \in \arg\max_{m' \in M(z)} \sum_{y \in Y} w(z, y) \Pr(y|\gamma^*(m')); \tag{5}$$

(iii) for any $m \in M$ and any $y \in S(\gamma^*(m))$:

$$y \in \arg\max_{y' \in Y} \sum_{z \in Z} v(z, y') \mathcal{P}^*(z|m); \tag{6}$$

(iv) the posterior $\mathcal{P}^*$ is derived consistently by using α_1^*, α_2^*, and β^* and the Bayes' rule whenever it is possible. Otherwise, $\mathcal{P}^*$ is some probability distribution over Z satisfying Requirement 1.

We assume the following tie-breaking rules: one for the voter and the other for the outlet.

Requirement 2 ***Tie-breaking rules***

(i) If the voter is indifferent between y_1 and y_2 under belief $\mathcal{P}^*(\cdot|m)$, then $\gamma^*(m) = (1/2, 1/2)$.

(ii) If the outlet observes policy pair z, such that $x_1 = x_2$, then $\beta^*(z) = z$.[22]

[22] To economize on notation, $\beta^*(z) = \{z\}$ is simply represented by $\beta^*(z) = z$.

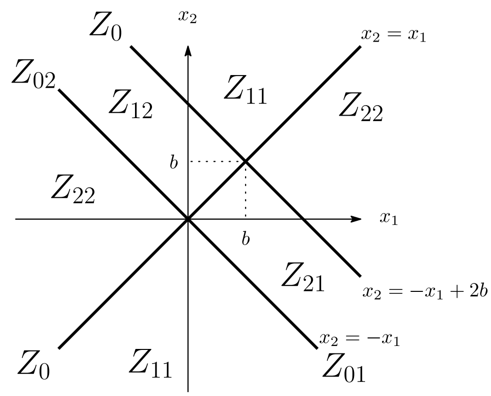


Figure 1: Distribution of preferences

In the subsequent analysis, we focus on PBEs where (i) the tie-breaking rules are satisfied, and (ii) the voter adopts undominated strategies. To simplify the referencing, an equilibrium in which the opportunistic candidate proposes x_1 and x_2 for certain is referred to as an (x_1, x_2) *equilibrium*.

We define the following notation and terminology. Let $Z(\alpha_1, \alpha_2) \equiv \{z \in Z \mid \Pr(z|\alpha_1, \alpha_2) > 0\}$ denote the set of possible policy pairs given strategies α_1 and α_2. Depending on the preferences defined by (2) and (3), the space of policy pairs Z is divided into the following regions, as shown in Figure 1. For $i, i', j, j' \in \{1, 2\}$, with $i \neq i'$ and $j \neq j'$:

$$\begin{aligned} Z_{ij} &\equiv \{z \in Z \mid v(z, y_i) > v(z, y_{i'}) \text{ and } w(z, y_j) > w(z, y_{j'})\}, \\ Z_{0j} &\equiv \{z \in Z \mid v(z, y_1) = v(z, y_2) \text{ and } w(z, y_j) > w(z, y_{j'})\}, \\ Z_0 &\equiv \{z \in Z \mid w(z, y_1) = w(z, y_2)\}. \end{aligned} \tag{7}$$

We refer to regions Z_{11}, Z_{22}, and Z_0 as *agreement regions* and to regions Z_{01}, Z_{02}, Z_{12}, and Z_{21} as *disagreement regions.* If a proposed policy pair lies in an agreement region, then the voter's and the outlet's preferences agree. In region Z_{11} (resp. Z_{22}), both the voter and the outlet strictly prefer y_1 (resp. y_2). In region Z_0, the outlet is indifferent between y_1 and y_2, while the voter could have a strict preference. On the contrary, if a proposed policy pair lies in a disagreement region, then the voter's and the outlet's preferences disagree. In regions Z_{12} and Z_{02} (resp. Z_{21} and Z_{01}), the voter has a weak preference for y_1 (resp. y_2), whereas the outlet strictly prefers y_2 (resp. y_1). Define $Z_j(\alpha_1, \alpha_2) \equiv Z_j \cap Z(\alpha_1, \alpha_2)$, for $j \in \{11, 22, 0, 01, 02, 12, 21\}$. For ease of reference, define $\bar{Z}_{12} \equiv Z_{12} \cup Z_{02}$ (resp. $\bar{Z}_{21} \equiv Z_{21} \cup Z_{01}$) and $\bar{Z}_{12}(\alpha_1, \alpha_2) \equiv \bar{Z}_{12} \cap Z(\alpha_1, \alpha_2)$ (resp. $\bar{Z}_{21}(\alpha_1, \alpha_2) = \bar{Z}_{21} \cap Z(\alpha_1, \alpha_2)$). Let $y^v(z)$ be the voter's ex post correct decision-making defined by:

$$y^v(z) \equiv \begin{cases} (1,0) & \text{if } |x_1| < |x_2|, \\ (1/2,1/2) & \text{if } |x_1| = |x_2|, \\ (0,1) & \text{if } |x_1| > |x_2|. \end{cases} \tag{8}$$

2.2 Discussion of the model

2.2.1 Unique voter

To simplify the analysis, we assume that the voter is unique. We can easily extend the model with unit mass voters, but we observe the same distortion mechanism as in the baseline model.[23] The unique voter in this model can be regarded as a representative swing voter. It is well known that swing voters have a major impact in determining the outcome of an election. Because swing voters change their minds according to the information they face, we can reasonably say that they are not partisan to particular parties and policies.[24] The voter in the model with ideal policy 0 is consistent with this interpretation.

2.2.2 Single media outlet with policy motivation

While the assumption of a single media outlet seems to be too demanding, this model is more applicable than it first appears. That is, the model with a single media outlet can be regarded as a reduced form of a model with multiple media outlets whose influence is imbalanced. In democratic countries, it is natural that ideologically different media outlets coexist; however, this does not necessarily mean that media coverage is balanced. Even in such countries, media coverage tends to be biased in one direction, and thus the minority may have less influence.[25] Note that the results in a multiple-outlet model with imbalanced influence are qualitatively the same as those in the single-outlet model. In that sense, we regard the single-outlet model as a reduced form of a more realistic setup. We revisit this point in Appendix B.3.1.

Given this interpretation, treating the single outlet as a particular profit-maximizing firm seems to be inappropriate. Instead, it should be viewed as a representative media outlet whose media bias reflects the aggregate tone of media coverage in that country. In other words, media bias b

[23] The detail is available from the author upon request.

[24] Campbell (2008) argues that "[t]hey (swing voters) are either moderates or people who are unable or unwilling to characterize their ideology."

[25] Empirical studies prove that media coverage is imbalanced even in the US. For example, Groseclose and Milyo (2005) find a strong liberal bias at the national level. Puglisi and Snyder (2015) also show a state-level imbalance on particular issues.

measures the extent to which conservative outlets have a stronger influence than the liberal outlets. The setup of policy-motivated outlets seems to be consistent with this interpretation.

2.2.3 Observability of the policies

Related to the observability of the policies, we add the following two restrictions: (i) candidates cannot send direct messages to voters, and (ii) only the outlet can observe the policies. These restrictions can be justified as follows. First, assumption (i) reflects the fact that active information acquisition is too costly for voters, as Downs (1957) argues. In reality, candidates have opportunities to send their own messages directly to voters (e.g., stump speeches). However, it seems to be too costly for most voters, especially swing voters, to actively acquire such information (e.g., attending stump speeches and directly asking questions). Thus, to save costs, rather than actively acquiring information, voters rely mainly on the news released by media outlets.

Second, assumption (ii) is included to simplify the analysis. As mentioned above, because the costs are sufficiently large, swing voters never actively acquire election-relevant information. By contrast, the costs might be low for ideological voters interested in elections, who might then acquire the information by themselves. However, we implicitly assume that their impact is negligible for the following reasons. First, these voters are in the minority, and as such their voting seems to have marginal impacts.[26] Second, while they might pass on the acquired information to the others through, for example, the social media, this information seems less credible because any fake news is not prohibited. That is, because such information tend to be ignored, we can say that the information from other than the mainstream media outlets seems to have only marginal impacts as long as the swing voters are rational.[27] For these reasons, we adopt this restriction to simplify the analysis.

2.2.4 Impossibility of fabrication

To capture the typical media manipulation, we assume that the news cannot be faked. As Groseclose and Milyo (2005) argue, media manipulation through the fabrication of information is less likely than manipulation by omission.[28] We then focus on the scenario where the outlet's reports must contain the truth for clarifying the impact of such a typical manipulation behavior of mass media.

[26] For example, according to Gottfried et al. (2016), only 1% of the voters actively acquired information in the 2016 presidential election.

[27] According to Allcott and Gentzkow (2017), the impact of fake news in the 2016 presidential election could be much smaller than Trump's margin of victory in the pivotal states, which is consistent with our argument.

[28] Groseclose and Milyo (2005) argue, "Instead, for every sin of commission, such as Glass or Blair, we believe that there are hundreds, and maybe thousands, of sins of omission–cases where a journalist chooses facts or stories that only one side of political spectrum is likely to mention."

2.2.5 Perfect commitment of the winning candidate

While the full commitment assumption of the winning candidate is demanding in our setup, we adopt it because of purely theoretical reasons. That is, we wish to separate the effects of media manipulation from those of imperfect commitment. For example, imperfect commitment to policy implementation can induce policy divergence, as noted by Banks (1990), Harrington (1992), and Callander and Wilkie (2007). However, the main purpose of this paper is to examine the effects of media manipulation on electoral competitions. To highlight the manipulation effects, we therefore include this extreme assumption.

2.2.6 Asymmetry between the candidates

We assume that the candidates are asymmetric in the sense that the proposed policies of ideological candidates are different, and $|r| < |l|$. This assumption is essential to the result: if candidates are completely symmetric, then policy convergence is more persistent. However, we do not require large asymmetry. It is sufficient to exclude completely symmetric scenarios to obtain the results. A detailed discussion appears in Appendix B.3.2.

2.2.7 Tie-breaking rule for the media outlet

We assume that the outlet discloses all information (i.e., $\beta(z) = z$) if the proposed policies converge. This assumption avoids a serious multiplicity of equilibria; that is, if $\beta^*(z) = Z$ holds for any convergent policy pair z, then any policy pair could be supported in equilibrium. However, if we require that the outlet fully discloses the true information with positive probability when the proposed policies converge, then such serious multiplicity disappears. Furthermore, we can show that the set of policy pairs that can be supported under the restriction is identical to that under the tie-breaking rule. That is, the tie-breaking rule is not crucial to the results. A detailed discussion is provided in Appendix B.3.3.

2.2.8 Nonstrategic ideological types

While nonstrategic ideological types seem to be essential to the results, this is merely for simplification and, thus, irrelevant to the results. That is, we can obtain similar results in the model in which any type of candidate is fully rational. The most important factor of this model is the voter's uncertainty about how candidates behave. For the detail, see in Appendix B.3.4.

Probability	(θ_1, θ_2)	Proposed policy pair	Winner	Equilibrium policy
p^2	(O, O)	$(0, 0)$	1 or 2	0
$p(1-p)$	(O, I)	$(0, l)$	1	0
$(1-p)p$	(I, O)	$(r, 0)$	2	0
$(1-p)^2$	(I, I)	(r, l)	1	r

Table 1: Equilibrium outcomes in the benchmark model

3 Benchmark: No Manipulation

In this section, we briefly review the model without media manipulation, as the benchmark. Because the voter always learns the true proposed policies, he can certainly cast the ballot for the candidate whose policy is closer to his ideal policy 0. Thus, as in standard Downsian models, the $(0, 0)$ equilibrium is the unique equilibrium in the benchmark model. Table 1 summarizes the equilibrium outcomes, showing that we can support the voter's ideal policy as the equilibrium policy unless both candidates are of the ideological type. The following proposition summarizes the result in the benchmark model.

Proposition 1 *Consider the benchmark model.*

(i) There exists a $(0, 0)$ equilibrium, and it is the unique equilibrium.

(ii) The voter's ideal policy is supported as the equilibrium outcome unless both candidates are of the ideological type.

4 Mechanisms of the Distortion

Now, we return to the model including media manipulation, which we refer to as the *manipulated news model*. In this section, we clarify the mechanism of the distortion. The equilibrium outcome is distorted compared with that in the benchmark model through the following two channels. The first is the distortion of the voter's behavior. That is, the outlet strategically suppresses the information about policies (*media manipulation*), and then the voter's decision-making could be incorrect ex post on the equilibrium path because of the remaining uncertainty about the proposed policies. The second is the distortion of the candidates' behaviors. That is, the candidates propose other policies than the voter's ideal policy in equilibrium to win the election by influencing the outlet's behavior through policy settings (*self-mediatization*). In other words, the interaction between media manipulation and self-mediatization distorts not only the information the voter receives, but also distorts the alternatives that he can choose. We refer to these as *direct distortion* and *indirect distortion*, respectively. As a result, there exist multiple equilibria including policy divergence, and the $(0,0)$ equilibrium does not always exist, which are the main differences to the benchmark model.

4.1 Direct distortion

First, we analyze a persuasion game between the outlet and voter given the candidates' proposed policies in the news-reporting stage. At the beginning of the news-reporting stage, the voter faces uncertainty about the proposed policy pair because of the uncertainty about the candidates' types. For example, suppose that $\alpha_1^* = \alpha_2^* = 0$ are the opportunistic-type candidates' equilibrium strategies. The voter then knows that either of the pairs in $Z(\alpha_1^*, \alpha_2^*) = \{(0,0), (0,l), (r,0), (r,l)\}$ is proposed in equilibrium, but he cannot specify which policy pair is actually proposed. In other words, given equilibrium strategies α_1^* and α_2^*, the voter, in equilibrium, could face uncertainty represented by a distribution over Z, whose support is $Z(\alpha_1^*, \alpha_2^*)$ at the beginning of the news-reporting stage.[29] Therefore, the news from the outlet is crucial for the voter to choose the correct candidate in this model. The following proposition states that media manipulation forces the voter's decision to be incorrect ex post even though the voter is fully rational.

Proposition 2 *Consider the manipulated news model.*

(i) There exists an equilibrium $(\alpha_1^*, \alpha_2^*, \beta^*, \gamma^*; \mathcal{P}^*)$ *such that* $\gamma^*(\beta^*(z)) = y^v(z)$ *for any* $z \in Z(\alpha_1^*, \alpha_2^*)$ *if and only if either (1)* $\bar{Z}_{12}(\alpha_1^*, \alpha_2^*) = \emptyset$ *or* $\bar{Z}_{21}(\alpha_1^*, \alpha_2^*) = \emptyset$ *holds or (2)* $Z_{02}(\alpha_1^*, \alpha_2^*) \neq \emptyset$, $Z_{01}(\alpha_1^*, \alpha_2^*) \neq \emptyset$, *and* $Z_{12}(\alpha_1^*, \alpha_2^*) = Z_{21}(\alpha_1^*, \alpha_2^*) = \emptyset$ *hold.*

(ii) In any equilibrium, there exists at least one policy pair $z \in Z$ *such that* $\gamma^*(\beta^*(z)) \neq y^v(z)$.

Intuitively, whether the voter's ex post correct decision-making is guaranteed on the equilibrium path depends on whether he can correctly infer the outlet's motivation behind the suppression.[30] Suppose, for example, that $\bar{Z}_{12}(\alpha_1^*, \alpha_2^*) \neq \emptyset$ and $\bar{Z}_{21}(\alpha_1^*, \alpha_2^*) = \emptyset$. In this scenario, the voter can correctly infer that the true policy pair is in $\bar{Z}_{12}$ after observing manipulated messages because only types in $\bar{Z}_{12}$ have an incentive to suppress information. Hence, the correct decision-making

[29] As long as we use the Nash concept, players correctly expect the strategies of others in equilibrium. In the manipulated news model, the policies proposed are the strategies of the candidates and, thus, the voter correctly expects the candidates' strategies in equilibrium. However, because the voter does not know the types of the candidates, he faces uncertainty about the proposed policy pair. For this reason, $p = 1$ is excluded.

[30] Proposition 2-(i) is a corollary of the well-known result in the literature on persuasion games. This is the necessary and sufficient condition for the existence of the *worst-case inference* for any message $m \in M$. See Giovannoni and Seidmann (2007), Hagenbach et al. (2014), and Miura (2014).

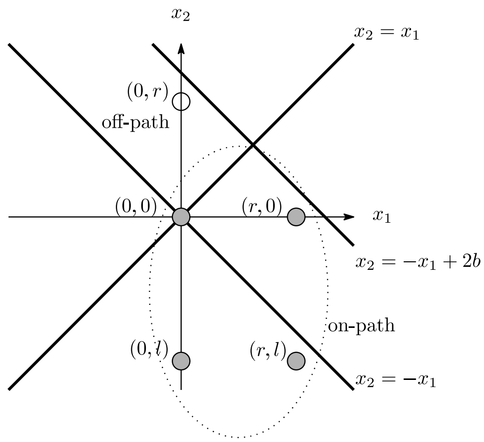


Figure 2: Incorrect decision-making for the off-the-equilibrium-path policy.

is guaranteed on the equilibrium path. However, if $Z_{12}(\alpha_1^*, \alpha_2^*) \neq \emptyset$ and $Z_{21}(\alpha_1^*, \alpha_2^*) \neq \emptyset$, then such an inference is impossible because two possibilities induce the suppression. Because of this indeterminacy, the voter's decision-making should be incorrect with positive probability on the equilibrium path, which is the direct distortion.

Even if the voter's decision-making is ex post correct on the equilibrium path, it must be incorrect at some off-the-equilibrium-path policy pair. Suppose, for example, that $b > r/2$, and fix $\alpha_1^* = \alpha_2^* = 0$. Because $\bar{Z}_{12}(\alpha_1^*, \alpha_2^*) = \emptyset$, as shown in Figure 2, there exists an equilibrium in the news-reporting stage in which the voter's ex post correct decision-making is guaranteed on the equilibrium path by Proposition 2-(i). To support this equilibrium, the voter's response to message $m' \equiv \{(0, r), (r, 0)\} \in M(r, 0)$ should be $\gamma^*(m') = (0, 1)$; otherwise, the outlet observing $z = (r, 0)$ deviates.[31] In this equilibrium, policy pair $z = (0, r)$, where the voter prefers candidate 1 but the outlet prefers candidate 2, is off the equilibrium path. Thus, given the voter's response $\gamma^*(m') = (0, 1)$, the outlet observing policy pair $z = (0, r)$ sends message m', and so candidate 2 definitely wins. That is, the voter's decision-making at policy pair $z = (0, r)$ is incorrect ex post.

In summary, the outlet successfully conceals some of the unfavorable information in any equilibrium. That is, the voter's decision-making is incorrect ex post. While incorrect decision-making on the equilibrium path distorts the equilibrium outcomes, it might seem that incorrect decision-making off the equilibrium path is irrelevant. However, even incorrect decision-making off the equilibrium path affects candidates' incentives significantly, as discussed next.

[31]To economize on notation, $M((x_1, x_2))$ is simply represented as $M(x_1, x_2)$.

Probability	Proposed policy pair	Media	Winner	Equilibrium policy
pq^2	$(0,0)$	discloses	1 or 2	0
$pq(1-q)$	$(0,r)$	suppresses	1 or 2	0 or r
$q(1-p)$	$(0,l)$	discloses	1	0
$pq(1-q)$	$(r,0)$	suppresses	1 or 2	0 or r
$p(1-q)^2$	(r,r)	discloses	1 or 2	r
$(1-q)(1-p)$	(r,l)	discloses	1	r

Table 2: Equilibrium outcomes in the mixed strategy equilibrium

4.2 Indirect distortion

Here, we analyze how opportunistic candidates behave. We note the following two main contrasts between the manipulated news and the benchmark models. First, there exist multiple equilibria including policy divergence equilibria. Second, the $(0,0)$ equilibrium does not exist unless the preference bias is small. For opportunistic candidates, the effective way of winning the election is altered in the manipulated news model because of the voter's incorrect decision-making, which is the indirect distortion. This is the origin of the contrasts.

The multiplicity of equilibria arises because proposing the voter's ideal policy becomes less attractive to the candidates owing to the voter's incorrect decision-making. In the benchmark model, appealing to the voter is the effective way in which to maximize the winning probability. That is, because the voter correctly recognizes the proposed policy pair, only a candidate who proposes a policy closer to the voter's ideal policy wins with positive probability. However, in the manipulated news model, the voter could not correctly recognize the attractiveness of the candidate who appeals to him because of media manipulation. As a result, the candidate who proposes a policy that is ex post less attractive to the voter could win with positive probability. Therefore, from the perspective of the candidates, proposing a policy other than the voter's ideal policy is not a bad idea. An example of an equilibrium other than $(0,0)$ is as follows, whose outcomes are summarized in Table 2.[32]

Claim 1 *Consider the manipulated news model, and suppose that $b > r$. Then, there exists an equilibrium where, for any $q \in (0,p)$, candidate 1 (resp. candidate 2) randomizes policies 0 and r with probabilities q/p (resp. q) and $1-q/p$ (resp. $1-q$), respectively.*

As demonstrated in this example, appealing to the voter may not be dominant for the candidates. Notice that the outlet suppresses information only when $x=(0,r)$ or $(r,0)$. Then, the voter

[32] "Discloses" and "suppresses" mean that the outlet sends $m=z$ and Z, respectively.

should be indifferent between y_1 and y_2 when manipulation is observed. That is, each candidate wins equally likely in this scenario. In this equilibrium, candidate 2's winning probability is $1/2$.[33] Now, we consider candidate 2's deviation to strategy $\alpha_2 = 0$. In the benchmark model, this deviation strictly improves his winning probability: because candidate 2 wins for certain under policy pair $z = (r, 0)$, his winning probability is $1 - q/2 > 1/2$ after this deviation.[34] Hence, such a mixed strategy equilibrium never exists in the benchmark model, as shown in Proposition 1. However, in the manipulated news model, this deviation does not strictly improve his winning probability. In this equilibrium, the outlet observing policy pair $z = (r, 0)$ successfully suppresses the information, and then the voter chooses candidate 1 with positive probability; that is, $\gamma^*(\beta^*(r, 0)) = (1/2, 1/2)$. Hence, candidate 2's winning probability does not change after this deviation. Because the voter's ex post incorrect decision-making for policy pair $z = (r, 0)$ makes appealing to the voter less attractive to candidate 2, an equilibrium other than $(0, 0)$ exists in the manipulated news model. It is worthwhile noting that policy divergence occurs on the equilibrium path.

The second contrast with the benchmark model is the fragility of the $(0, 0)$ equilibrium. The necessary and sufficient condition for the existence of the $(0, 0)$ equilibrium is as follows, which is the first main result of this paper.

Theorem 1 *Consider the manipulated news model. Then, there exists a $(0, 0)$ equilibrium if and only if either (i) $b \leq r/2$ holds or (ii) $b = r$ and $p \leq 1/2$ hold.*

As shown in Theorem 1, the $(0, 0)$ equilibrium does not exist when the preference bias is large. This fragility of the $(0, 0)$ equilibrium arises from the candidates' self-mediatization incentive. When information is suppressed, the voter could strictly prefer one candidate to the other. We refer to the preferred candidate as the *front-runner* and the less preferred candidate as the *underdog*. The front-runner then has an incentive to propose a policy likely to be suppressed to maintain his advantage under manipulation. On the contrary, the underdog has an incentive to propose a policy likely to be disclosed to mitigate the disadvantage under manipulation.[35] That is, the candidates try to win the election by influencing media coverage through policy setting. The incentives for such self-mediatization are the main force in breaking down the $(0, 0)$ equilibrium.

[33] From the perspective of candidate 2, the realized policy pairs are $(0, 0)$, $(0, r)$, $(r, 0)$, and (r, r). Notice that his winning probability under each policy pair is $1/2$ because each pair either (i) lies in 45° line or (ii) is suppressed.

[34] From the perspective of candidate 2, this deviation induces policy pairs $(0, 0)$ and $(r, 0)$ with probabilities q and $1 - q$, respectively. Because he wins with probabilities $1/2$ and 1 under $z = (0, 0)$ and $(r, 0)$, respectively, his winning probability is $1 - q/2$.

[35] The underdog behavior is associated with the Trump example mentioned above. In fact, Trump was not a prominent candidate in the early stage. For example, according to the poll conducted by Fox just after Trump's candidacy, Jeb Bush and Trump obtained 15 and 11 points, respectively.

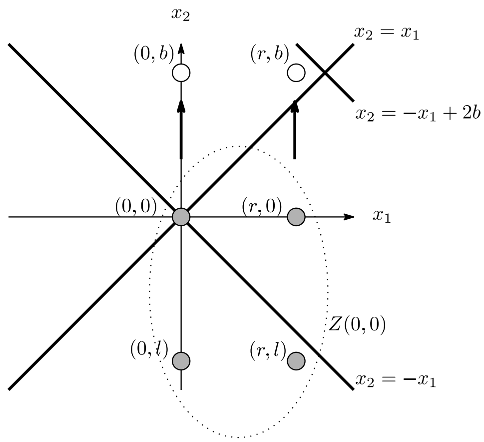


Figure 3: Corruption of the $(0,0)$ equilibrium by the self-mediatization incentive

Suppose, for example, that the $(0,0)$ equilibrium exists with $\gamma^*(\beta^*(r,0)) = (0,1)$ when $b > r$. Note that to support this equilibrium, $\gamma^*(m) = (0,1)$ should hold for any message $m \in M(r,0)$; otherwise, the outlet observing policy pair $z = (r,0)$ deviates. Because this message is also available to the outlet observing a policy pair in disagreement region $\bar{Z}_{12}$, $\gamma^*(\beta^*(z)) = (0,1)$ should hold for any $z \in \bar{Z}_{12}$; otherwise, the outlet sends a message including policy pair $z = (r,0)$. In this scenario, candidate 2 is the front-runner under messages including policy pair $z = (r,0)$, and he then has a strong self-mediatization incentive to encourage media manipulation to exploit the advantage. In other words, candidate 2 proposes a policy that is more likely to induce a policy pair that lies in disagreement region $\bar{Z}_{12}$ because he wins with certainty if such a policy pair is realized. Hence, candidate 2 deviates to strategy $\alpha_2 = b$ because the realized policy pair under this strategy certainly lies in disagreement region $\bar{Z}_{12}$, as shown in Figure 3. That is, the $(0,0)$ equilibrium collapses because of candidate 2's self-mediatization incentive. This collapse demonstrates that the voter's ex post incorrect decision-making off the equilibrium path affects the candidates' incentives.

In summary, the interaction between media manipulation and self-mediatization induces the direct and indirect distortion, which both distort the equilibrium outcomes. In addition to the direct distortion by the voter's incorrect decision-making, the equilibrium outcomes are indirectly distorted because the chosen policies are altered. That is, instead of appealing to the voter, the candidates exploit media manipulation to win the election. We can observe this distortion structure in any equilibrium except for the $(0,0)$ equilibrium. For example, in the mixed-strategy equilibrium specified in Claim 1, the direct distortion appears in the second and fourth rows of Table 2 (i.e., the voter chooses the unfavored candidate with positive probability). On the contrary, the indirect distortion appears in all rows of Table 2 in the sense that policy r is more likely to be proposed because of randomization. This is the distortion mechanism in the manipulated news model.

5 Set of Equilibria

We have thus far focused on the distortion mechanisms, showing that the best scenario for the voter never occurs when the media bias is sufficiently large. In this section, we analyze the next natural question: to what extent are the equilibrium outcomes distorted? First, we focus on a particular class of equilibria as an equilibrium selection, and then characterize the set of equilibria in terms of its degree of distortion, measured by the voter's ex ante expected utility. Second, we conduct comparative statics, which suggest that the outcomes become more dispersed as either the outlet becomes more biased or the candidates behave more opportunistically.

5.1 Equilibrium selection: Undominated simple equilibria

Let us introduce the following notation. We say that the outlet's strategy $\bar{\beta}$ is *simple* if it satisfies the following properties: (i) $S(\bar{\beta}(z)) \subseteq \{\{z\}, \bar{Z}_{12} \cup \bar{Z}_{21}\}$ for any $z \in Z$; and (ii) $\bar{\beta}(z) = z$ for any $z \in Z_0 \cup Z_{11} \cup Z_{22}$, and $\bar{Z}_{12} \cup \bar{Z}_{21}$ for any $z \in Z_{12} \cup Z_{21}$.[36] That is, the outlet fully discloses the information over the agreement region $Z_0 \cup Z_{11} \cup Z_{22}$, and suppresses it over disagreement region $Z_{12} \cup Z_{21}$.[37] Let B be the set of simple strategies of the outlet. To obtain clear results, we focus on the following equilibria.

Definition 2 ***Undominated simple equilibrium (hereafter, USE)***
A USE $(\alpha_1^*, \alpha_2^*, \beta^*, \gamma^*; \mathcal{P}^*)$ is a PBE satisfying the following conditions: (i) $\beta^* \in B$; and (ii) $(\alpha_1^*, \alpha_2^*) \in \Delta([0, b])^{*2}$.

Intuitively, a USE is a PBE constructed by a sense of weakly undominated strategies. In the standard definition of weak dominance, we require that a strategy is a (weak) best response to any strategies of the others. However, this requirement is too demanding in this environment. Because of the structures of spatial competition and costless message games, any strategies of the outlet and candidates are undominated, which makes the dominance criterion useless. Hence, when applying the weak dominance argument, we reasonably restrict the others' strategies, and focus on a PBE constructed by weakly undominated strategies in the above sense. A USE is defined based on this restriction.

Two remarks about the definition of the USE follow. First, we can restrict ourselves to simple strategies without loss of generality once we adopt this selection criterion. Let Γ be a set of the

[36] There is a degree of freedom for behaviors when the observed policy pair is in region $Z_{01} \cup Z_{02}$.

[37] Most of the existing studies applying persuasion games also focus on simple strategies. See, for example, Bernhardt et al. (2008) and Anderson and McLaren (2012).

voter's strategies such that a response to each message must be the ex post correct decision-making under some policy pair included in the observed message, defined by:

$$\Gamma \equiv \left\{ \gamma \in \Delta(Y)^M \,\middle|\, S(\gamma(m)) \subseteq \bigcup_{z \in m} S(y^v(z)) \text{ for any message } m \in M \right\}. \tag{9}$$

When applying the dominance argument to the outlet's strategies, the voter's available strategies are restricted to Γ. This restriction is reasonable because a rational voter never adopts strategies not included in Γ, and the outlet knows that the voter is rational. We can show that a simple strategy $\bar{\beta}$ is weakly undominated under this restriction. Furthermore, if we adopt a strong version of the PBE that requires as much consistency as possible, then we can show that any equilibrium outcome supported by the outlet's undominated strategies in the above sense can be replicated by an equilibrium with simple strategies. Thus, we can restrict our attention to equilibria with simple strategies.

Second, once the outcomes in the news-reporting stage are restricted to those induced by simple strategies, the set of weakly undominated strategies of the candidates should be $\Delta([0,b])^*$. Intuitively, on the one hand, proposing policy $x_i < 0$ (resp. $x_i > b$) is weakly dominated by proposing policy $x_i' = 0$ (resp. $x_i' = b$) because both the outlet and the voter agree to prefer x_i' to x_i. On the other hand, any strategy in $\Delta([0,b])^*$ is weakly undominated under that restriction. For example, suppose that $b \leq r/2$ and consider strategies $\alpha_1 = x_1 \in [0,b]$ and $\alpha_1' = x_1' \in X$ with $x_1' \neq x_1$. Notice that α_1 is not dominated by α_1' because if the others adopt strategies such that $\alpha_2 = x_1$, $\beta = \bar{\beta}$, $\gamma(x_1, x_1) = (1/2, 1/2)$ and $\gamma(\bar{Z}_{12} \cup \bar{Z}_{21}) = y_2$, then α_1 gives strictly higher winning probability to candidate 1 than under α_1'. Thus, the USE is less demanding than it looks. The formal proofs and a detailed discussion of this justification are provided in Appendix B.2.

5.2 Characterization of the equilibrium set

Hereafter, we focus on the USE, and characterize the equilibrium set in terms of the extent to which the equilibrium outcomes are distorted. We measure the distortion of the equilibrium outcomes by the voter's ex ante expected utility. The degree of distortion $d(e)$ in equilibrium $e = (\alpha_1^*, \alpha_2^*, \beta^*, \gamma^*; \mathcal{P}^*)$ is defined as follows:

$$d(e) = \left| \sum_{z \in Z} \sum_{m \in M} \sum_{y \in Y} v(z,y) \Pr(y|\gamma^*(m)) \Pr(m|\beta^*(z)) \Pr(z|\alpha_1^*, \alpha_2^*) \right| - (1-p)^2 r. \tag{10}$$

Note that $d(e) \geq 0$ for any USE e, and it is normalized to 0 under the $(0,0)$ equilibrium. Let $D(b,p)$ be the set of USE distortion levels under parameters b and p, and define $\underline{D}(b,p) \equiv \inf D(b,p)$ and $\overline{D}(b,p) \equiv \sup D(b,p)$, which represent the best- and worst-case scenarios for the voter, respectively. The next theorem characterizes the set of USEs, which is the second main result of this paper. We have the following subcases, depending on the magnitude of the media bias: (i) $0 < b \leq r/2$; (ii) $r/2 < b < r$; (iii) $r \leq b < |l|$; and (iv) $b \geq |l|$.

Theorem 2 *Consider the manipulated news model.*

(i) The infimum and supremum of the degree of distortion are as follows:

$$\underline{D}(b,p) = \begin{cases} 0 & \text{if } 0 < b \leq r/2, \\ p(1-p)(-r+2b) & \text{if } r/2 < b < r, \\ p(1-p)r & \text{if } b \geq r. \end{cases} \tag{11}$$

$$\overline{D}(b,p) = \begin{cases} p(2-p)b & \text{if } 0 < b < r, \\ pb + p(1-p)r & \text{if } r \leq b < |l|, \\ p|l| + p(1-p)r & \text{if } b \geq |l|. \end{cases} \tag{12}$$

(ii) For any b, p, there exists USE e such that $d(e) = d$ if and only if $\underline{D}(b,p) \leq d \leq \overline{D}(b,p)$, where the strict inequality holds if the minimum (resp. maximum) does not exist.

To characterize the equilibrium set, we basically construct a USE in which no front-runner exists. Notice that if each candidate is equally likely to win under suppression, then the candidates' deviations are easy to prevent. Suppose, for example, that both candidates adopt an identical strategy, and no front-runner exists. In this scenario, the proposed policies coincide with positive probability, in which each candidate's winning is equally likely. Hence, if the front-runner exists, then he deviates to a strategy inducing the suppression, as demonstrated in Theorem 1. However, because there is no front-runner, any deviation inducing policy pairs in the disagreement regions never improves his winning probability; that is, he obtains winning probability $1/2$ under any policy pair in the disagreement regions. Furthermore, deviation to a strategy inducing policy pairs in the agreement regions strictly decreases his winning probability because it requires that his proposed policy is farther from the voter's ideal policy than that of the opponent, and this information is fully disclosed. As a result, the candidates never deviate from such a symmetric strategy profile if no front-runner exists. By exploiting this property, we can construct a desired USE.

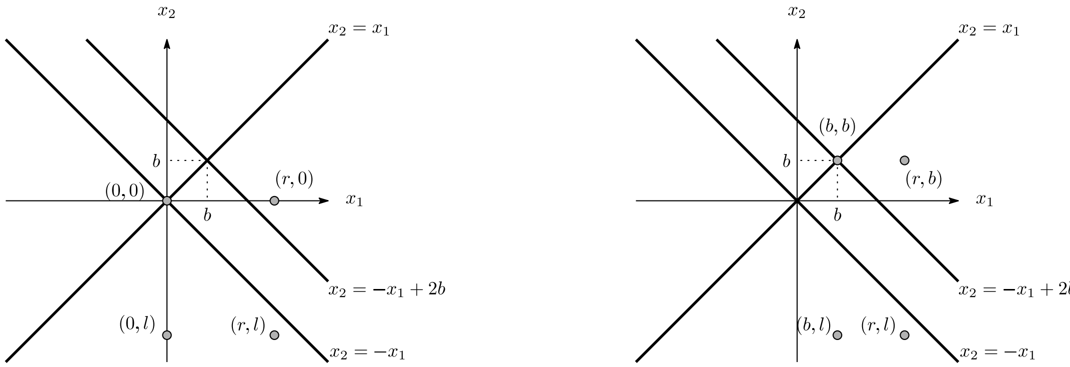


Figure 4: Case (i): infimum (left) and supremum (right)

In Case (i), the infimum and supremum of the distortion can be supported by the $(0,0)$ and (b,b) equilibria, respectively, and any value between the bounds can be supported by a symmetric USE. Because the media bias is sufficiently small, the proposed policy pairs are in the agreement regions if either of the candidates is ideological type. In other words, $\bar{Z}_{12}(\alpha_1,\alpha_2) \cup \bar{Z}_{21}(\alpha_1,\alpha_2) = \emptyset$ holds for any symmetric pure strategy profile, as demonstrated in Figure 4. Thus, because suppression only occurs off the equilibrium path, it is easy to construct a no-front-runner USE supporting such a symmetric pure strategy profile.

In Case (ii), the above argument can be partially applicable. On the one hand, we can construct a symmetric no-front-runner USE if the candidates adopt pure strategies $\alpha_1 = \alpha_2 \in [-r+2b, b]$. Because the disagreement regions are empty under such a strategy profile, as shown in Figure 5, we can easily construct a desired USE as in Case (i). Therefore, the (b,b) equilibrium attains the supremum, and the upper half of the equilibrium set is characterized. However, on the other hand, this argument cannot be used to characterize the lower half of the equilibrium set. If the candidates adopt pure strategies $\alpha_1 = \alpha_2 \in [0, -r+2b)$, then the disagreement regions are nonempty; that is, $\bar{Z}_{12}(\alpha_1,\alpha_2) = \emptyset$ and $\bar{Z}_{21}(\alpha_1,\alpha_2) \neq \emptyset$, and then candidate 2 is the front-runner. Hence, such a symmetric strategy profile cannot be supported in equilibrium because either of the candidates has an incentive to deviate, as demonstrated in Theorem 1. We face the same problem if candidate 2 proposes policy $x_2 \in [0, -r+2b)$ with positive probability.

In contrast to the previous scenarios, we construct a USE where candidate 1 is the front-runner to characterize the lower half of the equilibrium set. Suppose, for example that $\alpha_1 = 0$ and $\alpha_2 = -r+2b$ are chosen when $p \leq 2/3$.[38] As shown in Figure 5, candidate 1 has no incentive to deviate because he wins for certain as the front-runner. While candidate 2 appears to deviate to $\alpha_2 = 0$ to induce policy convergence, he does not. Because candidate 1 is more likely to be

[38]Even if $p > 2/3$, we can construct a similar USE. See Lemma 6 for the details.

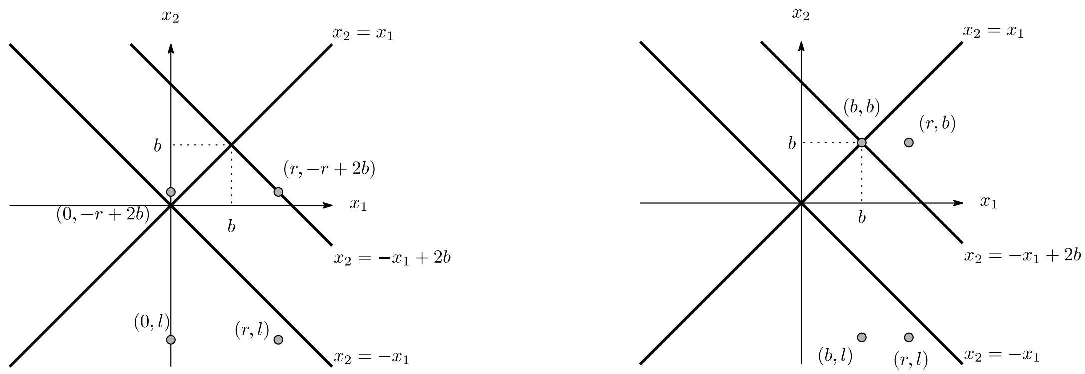


Figure 5: Case (ii): infimum (left) and supremum (right)

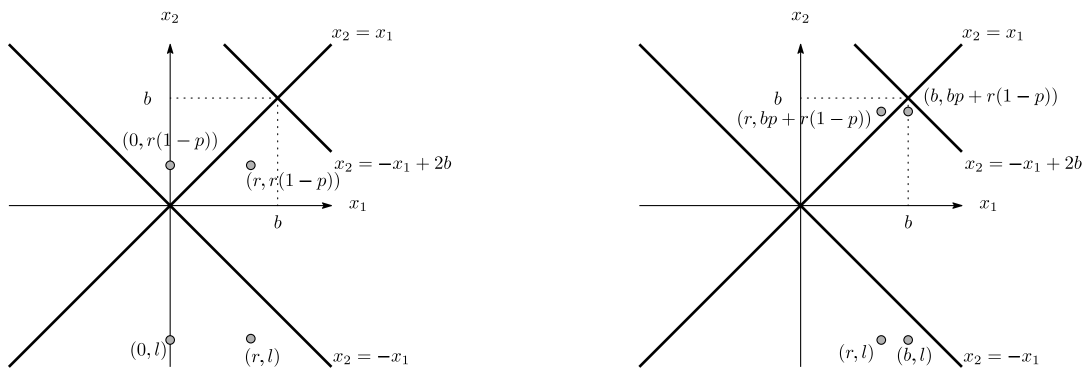
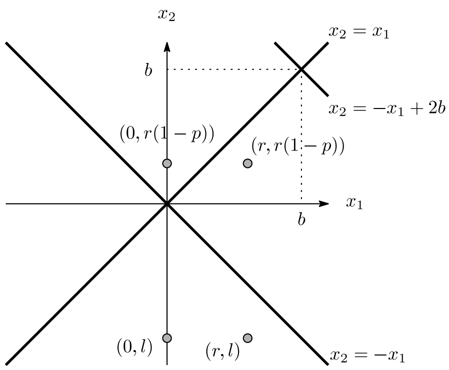


Figure 6: Case (iii): infimum (left) and supremum (right)

the ideological type, a victory over the ideological candidate 1 is more beneficial for candidate 2 compared with a tie with opportunistic candidate 1. Thus, this strategy profile is supported in a USE that attains the infimum. Because we can construct a similar USE where $\alpha_1 \in (0, -r+2b)$ and $\alpha_2 = -r+2b$, the lower half of the equilibrium set is also characterized.

In Cases (iii) and (iv), each distortion level is supported by a no-front-runner USE, where the candidates adopt different strategies. Because the media bias is sufficiently large, the disagreement regions are nonempty even if the candidates adopt an identical strategy. To construct the desired USE, candidate 1's conditional expected policy, given that the realized policy pair is in the disagreement regions, should be equivalent to that of candidate 2, as shown in Figures 6 and 7. In other words, the following strategy profile can construct a no-front-runner USE: $\alpha_1 = x_1$ and $\alpha_2 = px_1 + (1-p)r$ for $x_1 \in [0, b]$ in Case (iii), and $[0, |l|)$ in Case (iv). By continuously changing x_1, we can obtain that characterization.

The existence of continuum equilibria is in sharp contrast to previous studies. This property comes from the fact that the outlet sends the same (nondisclosure) message over all disagreement regions. That is, the voter's reaction is not sensitive to changes in the policy pair as long as it lies



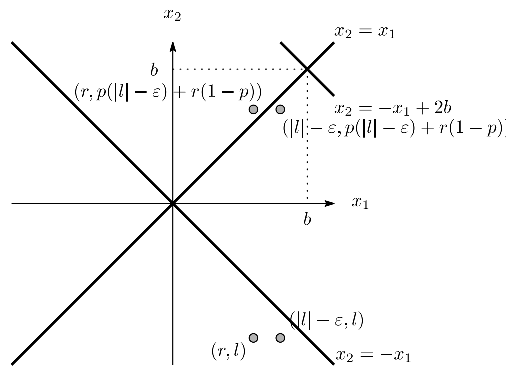


Figure 7: Case (iv): infimum (left) and supremum (right)

in the disagreement regions, because his observation does not vary. As a result, many policy pairs can be supported in the USE.

5.3 Comparative statics

In this subsection, we establish the comparative statics on the equilibrium set owing to changes in b (i.e., the degree of media bias), and p (i.e., the likelihood of being an opportunistic type). Let $\pi(b, p) \equiv \overline{D}(b, p) - \underline{D}(b, p)$ be the size of the equilibrium set. Theorem 2 implies that for any $b > 0$ and $p \in (0, 1)$, the size of equilibrium set $\pi(b, p)$ is as follows:

$$\pi(b, p) = \begin{cases} p(2-p)b & \text{if } 0 < b \leq r/2, \\ p^2 b + p(1-p)r & \text{if } r/2 < b < r, \\ pb & \text{if } r \leq b < |l|, \\ p|l| & \text{if } b \geq |l|. \end{cases} \tag{13}$$

We can observe the monotonicity of the equilibrium set in terms of the media bias, as shown in Figure 8, the shaded regions of which represent the equilibrium set.

Corollary 1 *Consider the manipulated news model.*

(i) For any $p \in (0, 1)$, both $\underline{D}(b, p)$ and $\overline{D}(b, p)$ are nondecreasing in b.

(ii) For any $p \in (0, 1)$, $\pi(b, p)$ is nondecreasing in b.

The distortion of the equilibrium outcomes becomes more severe as the outlet becomes more biased. First, both the infimum and the supremum of the degree of distortion are weakly increasing in b (i.e., the voter's welfare never improves as b increases). While the upper and lower bounds monotonically change, these behaviors are different, as shown in Figure 8. The infimum is constant

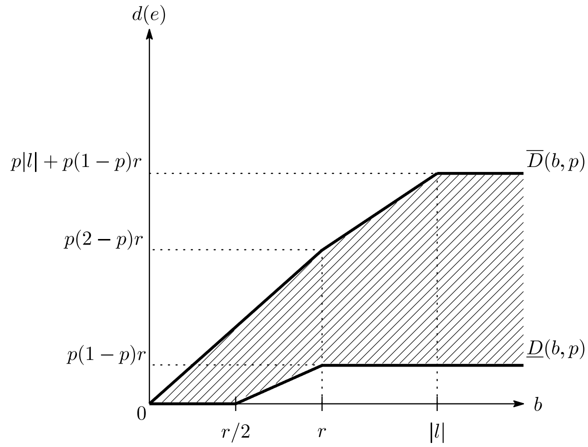


Figure 8: Comparative statics in b

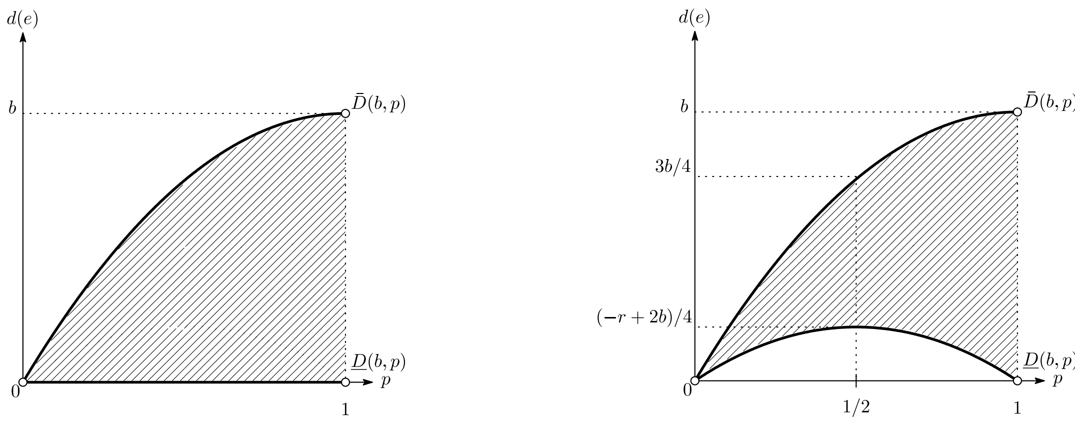


Figure 9: Comparative statics in p: Case (i) (left) and Case (ii) (right)

in the bias except for $r/2 < b \leq r$. That is, the best scenario for the voter is not to be sensitive to the bias unless it is intermediate. On the contrary, the worst scenario for the voter is being sensitive to the bias (i.e., the supremum is strictly increasing in b up to $|l|$). Second, the equilibrium set is expanding in b. That is, the equilibrium outcomes are more dispersed as the outlet becomes more biased, which comes from the fact that the more extreme policies are suppressed as the outlet becomes more biased.

In contrast to the above, the infimum may not be monotonic in the likelihood of the opportunistic type, but the equilibrium set has a similar monotonicity in p, as shown in Figure 9, the shaded region of which represents the equilibrium set.[39]

[39]We can draw similar figures for Cases (iii) and (iv), as in Case (ii).

Corollary 2 *Consider the manipulated news model.*

(i) For any $b > 0$, $\overline{D}(b, p)$ is strictly increasing in p.

(ii) $\underline{D}(b, p)$ is constant in p if $0 < b < r/2$; otherwise, it is nonmonotonic in p.

(iii) For any $b > 0$, $\pi(b, p)$ is strictly increasing in p.

Intuitively, the main source of the distortion of the supremum is that opportunistic candidates choose extreme policies. Then, the supremum is strictly increasing in p because the impact of these extreme policies grows as the candidates become more opportunistic. By contrast, the behavior of the infimum is more complicated. As shown in Theorem 1, if the media bias is sufficiently small, then the $(0, 0)$ equilibrium exists irrespective of p; that is, no distortion appears for any p in Case (i). However, the $(0, 0)$ equilibrium no longer exists when the bias is not as small. In Cases (ii) to (iv), the main source of the distortion of the infimum comes from the scenario in which candidates 1 and 2 are ideological and opportunistic, respectively. Note that because this scenario occurs with probability $p(1 - p)$, the infimum is nonmonotonic in p, as shown in Figure 9.[40] That is, it is maximized at $p = 1/2$, and converges to 0 as $p \to 1$ because this scenario becomes less likely. As a result, the equilibrium outcomes are more dispersed as the likelihood of the opportunistic type increases.

6 Misspecification under Reduced Form Models

In this section, we discuss the misspecification of the distortion caused by simplifications. As mentioned in Section 1, most existing works omit either competition among the candidates or strategic media manipulation. While this is a useful simplification, it ignores the interaction of these aspects, which is crucial for determining the distortion level, as demonstrated thus far. Hence, we discuss the following question: to what extent do the simplifications misspecify the distortion? To answer this question, we, hereafter, consider the *no competition* and *nonstrategic outlet models*, in which behaviors of the candidates and outlet are exogenously fixed, respectively. We then discuss the extent to which the distortion is misspecified by comparing with the manipulated news model. The conclusion is that the misspecification could be nonnegligible.

[40] While the nonmonotonicity might collapse if the probability of being the opportunistic type is different among the candidates, qualitatively the same results can be obtained.

6.1 No competition model

To clarify the impact of competition among the candidates, we consider the model in which candidate 1 is not a player. That is, we assume that if candidate 1 is the opportunistic type, then he chooses policies following nondegenerate distribution $\alpha_1 \in \Delta^*([0,b])$ that is exogenously given. The remaining setup is equivalent to that in the baseline model, and we focus on USEs.

Here, the degree of distortion is highly sensitive to the detail of the setup. To demonstrate this, consider the scenario in which $r \le b < |l|$, and $\alpha_2 = x_2 \in [0,b]$ is also exogenously fixed.[41] That is, the indirect distortion is exogenously fixed in this scenario. Hence, the degree of distortion is uniquely determined as long as we focus on USEs.[42] Let $\hat{x}_1 \equiv \sum_{x_1 \in X} x_1 \alpha_1(x_1)$ and $\tilde{x}_1 \equiv p\hat{x}_1 + (1-p)r$ be candidate 1's expected policy conditional on being the opportunistic type, and his ex ante expected policy, respectively. Notice that the voter's response to the suppressed news $m = \bar{Z}_{12} \cup \bar{Z}_{21}$ is choosing y_1 (resp. y_2) if $\tilde{x}_1 < x_2$ (resp. $\tilde{x}_1 > x_2$), and then we have the following claim.

Claim 2 *Consider the no competition model with $r \le b < |l|$ and $\alpha_2 = x_2$ being exogenously fixed. Then, the degree of distortion $d(\alpha_1, x_2)$ is given by:*

$$d(\alpha_1, x_2) = \begin{cases} px_2 + p(1-p)\hat{x}_1 & \text{if } x_2 \le \tilde{x}_1, \\ p\hat{x}_1 + p(1-p)r & \text{otherwise.} \end{cases} \tag{14}$$

Compared with Theorem 2, the dispersion of the equilibrium outcomes disappears; however, the degree of distortion is crucially dependent on α_1 and x_2, as shown in Figure 10. In the literature, fixed policy pairs are frequently assumed, and then Claim 2 suggests that a welfare analysis based on this simplification might over- or underestimate the loss depending on the detail.

An important observation is that while distortion $d(\alpha_1, x_2)$ never exceeds the supremum $\overline{D}(b,p)$ of the baseline model, it can be lower than its infimum $\underline{D}(b,p)$ depending on α_1 and x_2. In other words, the distortion could be underestimated in the no competition model. Furthermore, no distortion is approximately attainable as α_1 and x_2 goes to 0, which demonstrates the sharp contrast with the baseline model in which there exists no $(0,0)$ equilibrium. While this implication may be less convincing because we can arbitrarily choose policy pairs, we can insist that the same

[41] Duggan and Martinelli (2011) adopt a similar setup.

[42] Even if we focus on simple strategies, there is a degree of freedom on the behavior when $z \in Z_{01} \cup Z_{02}$. However, because the voter is indifferent between y_1 and y_2 under these policy pairs, his decision never affects the degree of distortion.

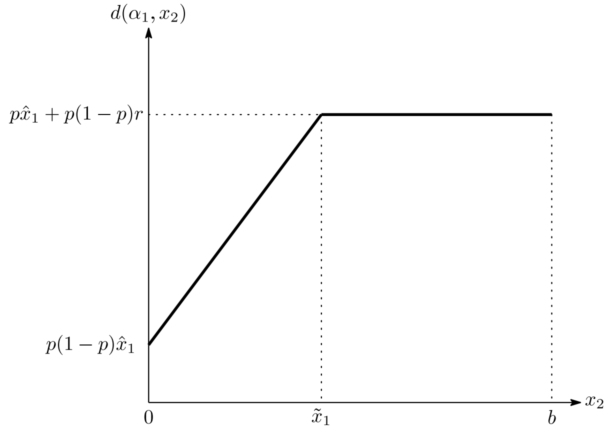


Figure 10: No competition model

observation is obtained even if α_2 is endogenously determined in equilibrium, as shown in the following proposition. In other words, the one-sided indirect distortion is insufficient to prevent the underestimation. Let $D_{nc}(\alpha_1, p)$ be the set of the equilibrium distortion in the no competition model, which is characterized as follows.

Proposition 3 *Consider the no competition model with $r \leq b < |l|$.*

(i) The set of the equilibrium distortion is characterized as follows:

$$D_{nc}(\alpha_1, p) = \begin{cases} [p(1-p)\hat{x}_1, p\hat{x}_1 + p(1-p)r] & \text{if } \hat{x}_1 \neq 0, \\ (0, p(1-p)r] & \text{otherwise.} \end{cases} \tag{15}$$

(ii) For any $\alpha_1 \in \Delta([0,b])^*$, p, and $d \in D_{nc}(\alpha_1, p)$, $d \leq \overline{D}(b, p)$ holds.

(iii) If $\alpha_1 = 0$, then for any $\varepsilon > 0$, there exists an equilibrium whose distortion is less than ε.

The lack of competition among the candidates makes the equilibrium outcomes more dispersed in the sense that the distortion level never exceeds the upper bound $\overline{D}(b, p)$ but can be less than the lower bound $\underline{D}(b, p)$. Intuitively, this misspecification comes from the fact that we can ignore candidate 1's incentive. In other words, candidate 2's equilibrium strategy is not restricted by the incentive condition of candidate 1. From the perspective of the voter, candidate 1's expected policy conditional on the suppression is greater than $(1-p)r$ in the baseline model. Hence, to support an equilibrium, the expected policy of candidate 2 also cannot be lower than $(1-p)r$; otherwise, candidate 2 becomes the front-runner, which induces candidate 1's deviation. However, it is unnecessary to be concerned about candidate 1's incentive in the no competition model. That is, even if candidate 2's expected policy is less than $(1-p)r$, it can be supported in equilibrium. As a result, the equilibrium could be less distorted.

On the contrary, because candidate 1's incentive is not binding in the most distorted equilibrium of the baseline model, it is also attainable even in the no competition model. In the upper bound equilibrium constructed in Theorem 2, candidate 1's expected policy is maximally distorted. Hence, to exceed that bound, candidate 2's expected policy conditional on the suppression must be more than $pb+(1-p)r$, and the voter chooses y_2. However, this is impossible. If candidate 2's expected policy is greater than that value, then the voter chooses y_1. Indeed, this argument remains valid irrespective of candidate 1's incentive conditions. Therefore, the distortion is not overestimated.[43] This result suggests that the competition among the candidate shrinks the equilibrium set, allowing us to conclude that the lack of competition could underestimate the distortion.[44]

6.2 Nonstrategic outlet model

Next, we discuss the impact of strategic media manipulation by investigating the model in which the media outlet is nonstrategic. That is, we assume that the outlet suppresses information with probability $q \in (0,1)$, and fully discloses it with the remaining probability under any policy pair.[45] The message space $M(z)$ is redefined as $M(z) \equiv \{z, \phi\}$ for any $z \in Z$, where ϕ represents the suppressed message. Except for this modification, the setup is identical to the baseline model, and we focus on the PBEs in which $\alpha_i \in \Delta^*([0,b])$ to ensure a fair comparison. We want to emphasize that, in this modified scenario, there exists a unique equilibrium, and then the distortion is either over- or underestimated, as shown in the following proposition.

Proposition 4 *Consider the nonstrategic outlet model.*

(i) There exists a $(0,0)$ *equilibrium in which candidate* 1 *is the front-runner, and it is the unique equilibrium.*

(ii) The degree of distortion is $d(q) = p(1-p)qr$.

The uniqueness of the $(0,0)$ equilibrium is in sharp contrast to the baseline model because of stochastic media manipulation; that is, any information is disclosed with positive probability. In the baseline model, the outlet's equilibrium behavior is deterministic. Hence, if each candidate wins

[43] The conclusion depends on the media bias in the baseline model. The detail is in Appendix B.4.1.
[44] Even if we fix candidate 2's strategy, we can obtain a similar underestimation. The detail is in Appendix B.4.2.
[45] Adachi and Hizen (2014) and Pan (2014) adopt a similar (but more general) setup to represent media manipulation.

equally likely under suppression, he has no incentive to deviate because his winning probability is not sensitive to deviations. By exploiting this no-front-runner structure, multiple outcomes can be supported in equilibrium, as mentioned above. However, in the nonstrategic outlet model, the candidates' winning probabilities are more sensitive to deviations because any information is disclosed with positive probability. In other words, if the information is disclosed, only the candidate who proposes a policy closer to the voter's ideal policy wins with positive probability, as in the standard Downsian model. Therefore, because each candidate focuses on the event where the information is disclosed to increase his winning probability, his incentive for appealing to the voter is stronger than the self-mediatization incentive. As a result, the uniqueness of the $(0, 0)$ equilibrium appears.

Proposition 4 insists that the degree of distortion is over- or underestimated in the nonstrategic outlet model. Notice that only the direct distortion matters in this scenario. In particular, the outcomes are distorted by the voter's incorrect decision-making when only candidate 1 is the ideological type and the information is suppressed. This distortion could be over- or underestimated depending on the media bias. To demonstrate this, we compare the degree of distortion in the nonstrategic outlet model with that in the lower bound of the baseline model.[46] If the bias is small (i.e., $0 < b \leq r/2$), then the distortion is overestimated. In the baseline model, the lower bound is given by the $(0, 0)$ equilibrium, and policy pair $z = (r, 0)$ is in the agreement regions. Hence, this information is certainly disclosed, and then the voter chooses the correct candidate for certain. However, in the nonstrategic outlet model, this information is suppressed with positive probability, which implies the voter's incorrect decision-making. As a result, the distortion is exaggerated. On the contrary, if the bias is large (i.e., $b \geq r$), then the distortion is underestimated for the converse reason. In the baseline model, the lower bound is attainable by the $(0, (1-p)r)$ equilibrium, and policy pair $z = (r, (1-p)r)$ is in the disagreement regions. Because this information is certainly suppressed, the voter chooses the incorrect candidate for certain.[47] However, the associated policy pair is disclosed with positive probability in the nonstrategic outlet model, which implies the voter's correct decision-making. As a result, the distortion is mitigated.

As a final remark, we can observe a similar misspecification even if we relax the assumption that the manipulation probability is identical among the policy pairs. Suppose that the outlet suppresses the information with probability $q_1 \in (0, 1)$ (resp. $q_2 \in (0, 1)$) if the policy pair is in the agreement

[46] The detailed analysis is in Appendix B.5.1.

[47] In fact, we construct a $(0, (1-p)r)$ equilibrium with no front-runner. However, because the voter is indifferent under suppression, the degree of distortion is equivalent even if it is regarded as an equilibrium where candidate 1 is the front-runner. We adopt this interpretation to clarify the difference.

(resp. disagreement) regions, and $q_1 < q_2$. If the difference between q_1 and q_2 is not large, then the same intuition mentioned above holds. On the contrary, if the difference is large, then the scenario becomes closer to the baseline model, which induces multiple equilibria. However, because the equilibrium policies are bounded above by r, the upper bound of the distortion is underestimated when the bias is not small. The detailed analysis of the relaxed model is in Appendix B.5.2.[48]

7 Conclusion

This paper investigates the interaction between media manipulation and self-mediatization in elections by analyzing the manipulated news model, where the candidates, outlet, and voter are all rational, and their behaviors are determined endogenously. First, we specify a mechanism that distorts the equilibrium outcomes (i.e., direct and indirect distortion). In the manipulated news model, the voter's decision-making could be incorrect, even though the voter is fully rational (direct distortion). Hence, appealing to the voter is less attractive to the candidates. Instead, the candidates have an incentive to win the election by exploiting media coverage (indirect distortion). As a result, there exist multiple equilibria; however, policy convergence to the voter's ideal policy cannot be supported in equilibrium when the outlet is sufficiently biased.

Second, we focus on the USE, which is a PBE constructed from undominated strategies under the restriction, and characterize the set of USEs in terms of the degree of distortion, measured by the voter's ex ante expected utility. In contrast to previous studies, the equilibrium outcomes are significantly dispersed in that there exist continuum equilibria. The comparative statics suggest that the equilibrium outcomes are more dispersed as either the outlet becomes more biased or the candidates behave more opportunistically.

Finally, we demonstrate that omitting either competition among the candidates or strategic media manipulation as a simplification could nonnegligibly misspecify the severity of the distortion. Because we can ignore the incentive compatibility of nonstrategic candidates, the no competition model approximately achieves the first-best outcome, which underestimates the distortion. Likewise, because the proposed policies are disclosed with positive probability in the nonstrategic outlet model, the indirect distortion is drastically mitigated. As a result, this simplification could either over- or underestimate the distortion depending on the media bias. In conclusion, we emphasize the importance of the explicit modeling of these factors to ensure the fair assessment of media manipulation.

[48] If the difference between q_1 and $_2$ is intermediate, then we face the nonexistence of equilibrium, which comes from the nonexistence of an equilibrium with no front-runner.

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Appendix A: Proofs

In this appendix, we provide the proofs of the main results. The omitted proofs for minor results are in Appendix B.

A.1 Preliminary

Notice that the voter's equilibrium strategy γ^* and posterior belief $\mathcal{P}^*(\cdot|m)$ for on-the-equilibrium-path message m are the same structure in any equilibrium. Given equilibrium strategies α_1^*, α_2^* and β^*, the posterior belief for on-the-equilibrium-path message m is determined by Bayes' rule as follows:

$$\mathcal{P}^*(z|m) = \begin{cases} \dfrac{\beta^*(m|z)\Pr(z|\alpha_1^*, \alpha_2^*)}{\displaystyle\sum_{z' \in Z(\alpha_1^*, \alpha_2^*)} \beta^*(m|z')\Pr(z'|\alpha_1^*, \alpha_2^*)} & \text{if } z \in Z(\alpha_1^*, \alpha_2^*), \\ 0 & \text{otherwise.} \end{cases} \tag{A.1}$$

Given posterior belief $\mathcal{P}^*$, the voter's undominated equilibrium strategy is uniquely determined as follows:

$$\gamma^*(m) = \begin{cases} (1,0) & \text{if } \mathbb{E}[|x_1||m] < \mathbb{E}[|x_2||m], \\ (1/2, 1/2) & \text{if } \mathbb{E}[|x_1||m] = \mathbb{E}[|x_2||m], \\ (0,1) & \text{if } \mathbb{E}[|x_1||m] > \mathbb{E}[|x_2||m]. \end{cases} \tag{A.2}$$

Hereafter, to avoid trivial repetition, we omit the description of the voter' equilibrium strategy and on-the-equilibrium-path beliefs. Furthermore, to simplify the representation, we characterize off-the-equilibrium-path beliefs by their supports. Any probability distribution with specified support is compatible with the equilibrium strategies.

Let $\mu_i(z)$ be candidate i's winning probability under policy pair z.[49] Let $U_i(\alpha_1, \alpha_2, \beta, \gamma)$ be candidate i's expected utility given α_1, α_2, β, and γ defined by:

$$U_i(\alpha_1, \alpha_2, \beta, \gamma) \equiv \sum_{z \in Z} \sum_{m \in M} u_i(y_i)\Pr(y_i|\gamma(m))\Pr(m|\beta(z))\Pr(z|\alpha_1, \alpha_2, \theta_i = O). \tag{A.3}$$

A.2 Proof of Proposition 2

(i) (Necessity) Suppose, in contrast, that there exists equilibrium $(\alpha_1^*, \alpha_2^*, \beta^*, \gamma^*; \mathcal{P}^*)$ such that $\gamma^*(\beta^*(z)) = y^v(z)$ for any $z \in Z(\alpha_1^*, \alpha_2^*)$ when either (1) $Z_{02}(\alpha_1^*, \alpha_2^*) \neq \emptyset$ and $Z_{21}(\alpha_1^*, \alpha_2^*) \neq \emptyset$ or (2) $Z_{12}(\alpha_1^*, \alpha_2^*) \neq \emptyset$ and $\bar{Z}_{21}(\alpha_1^*, \alpha_2^*) \neq \emptyset$ holds.[50] For Case (1), let $z \in Z_{02}(\alpha_1^*, \alpha_2^*)$ and $z' \in Z_{21}(\alpha_1^*, \alpha_2^*)$, and then $\gamma^*(\beta^*(z)) = (1/2, 1/2)$ and $\gamma^*(\beta^*(z')) = (0, 1)$ holds. To be incentive compatible, the following conditions must be satisfied: (a) $\gamma_1^*(m) \geq 1/2$ for any message $m \in M(z)$, and (b)

[49] Although the winning probabilities depend on the outlet's and the voter's strategies, we omit the explicit description of these factors for saving notation if it is not confusing.

[50] To simplify the notation, we, hereafter, use this representation even when the outlet adopt mixed strategies.

$\gamma_1^*(m') = 0$ for any message $m' \in M(z')$; otherwise, the outlet that observes either policy pair z or z' has an incentive to deviate from the equilibrium strategy. However, there is no incentive compatible reaction to message $m = \{z, z'\} \in M(z) \cap M(z')$, which is a contradiction. We can derive a contradiction in Case (2) by the similar argument.

(Sufficiency) Consider Case (1). Without loss of generality, we assume that $\bar{Z}_{12}(\alpha_1^*, \alpha_2^*) = \emptyset$. We then show that the following is a desired PBE in the news-reporting stage:

$$\beta^*(z) = \begin{cases} \bar{Z}_{12} \cup Z_{21} & \text{if } z \in \bar{Z}_{12} \cup Z_{21}, \\ z & \text{otherwise.} \end{cases} \tag{A.4}$$

$$S(\mathcal{P}^*(\cdot|m)) \subseteq \begin{cases} m \cap Z_{21} & \text{if } m \cap Z_{21} \neq \emptyset, \\ m \cap Z_{01} & \text{if } m \cap Z_{21} = \emptyset \text{ and } m \cap Z_{01} \neq \emptyset, \\ m & \text{otherwise.} \end{cases} \tag{A.5}$$

Note that $\gamma^*(\bar{Z}_{12} \cup Z_{21}) = (0, 1)$ and $\gamma^*(\{z\}) = y^v(z)$ given posterior $\mathcal{P}^*$.[51] We then check the optimality of the outlet's equilibrium strategy β^* given the voter's equilibrium strategy γ^*. If $z \notin \bar{Z}_{21}$, then the outlet has no incentive to deviate from $\beta^*(z)$ because it induces her preferred policy for certain. If $z \in \bar{Z}_{21}$, then the outlet also has no incentive to deviate because she cannot induce her preferred policy with positive probability. Thus, β^* is the outlet's best response. Finally, note that if $z \in Z(\alpha_1^*, \alpha_2^*) \backslash Z_{21}(\alpha_1^*, \alpha_2^*)$, then $\gamma^*(\beta^*(z)) = y^v(z)$ holds because the outlet fully discloses the information. If $z \in Z_{21}(\alpha_1^*, \alpha_2^*)$, then $\gamma^*(\beta^*(z)) = (0, 1) = y^v(z)$ holds. Therefore, this is a desired PBE.

Consider Case (2). We show that the following is a desired PBE in the news-reporting stage:

$$\beta^*(z) = \begin{cases} \bar{Z}_{12} \cup \bar{Z}_{21} & \text{if } z \in \bar{Z}_{12} \cup \bar{Z}_{21}, \\ z & \text{otherwise.} \end{cases} \tag{A.6}$$

For off-the-equilibrium-path message m such that $m \cap \bar{Z}_{12} \neq \emptyset$ and $m \cap \bar{Z}_{21} \neq \emptyset$:

$$\mathcal{P}^*(\tilde{z}|m) = \begin{cases} a & \text{if } \tilde{z} = z, \\ 1 - a & \text{if } \tilde{z} = z', \\ 0 & \text{otherwise,} \end{cases} \tag{A.7}$$

where $z = (x_1, x_2) \in m \cap \bar{Z}_{12}$, $z' = (x_1', x_2') \in m \cap \bar{Z}_{21}$, and $a \equiv (|x_1'| - |x_2'|)/(|x_1'| - |x_2'| + |x_2| - |x_1|)$

[51] Message $m = \bar{Z}_{12} \cup Z_{21}$ is sent on the equilibrium path if $Z_{21}(\alpha_1^*, \alpha_2^*) \neq \emptyset$. Note that, in this scenario, $S(\mathcal{P}^*(\cdot|\bar{Z}_{12} \cup Z_{21})) \subset Z_{21}$ holds on the equilibrium path because $\bar{Z}_{12}(\alpha_1^*, \alpha_2^*) = \emptyset$.

whenever it is well-defined; otherwise, a is any value in $[0,1]$.[52] For other off-the-equilibrium-path messages:

$$S(\mathcal{P}^*(\cdot|m)) \subseteq \begin{cases} m \cap \bar{Z}_{12} & \text{if } m \cap \bar{Z}_{12} \neq \emptyset \text{ and } m \cap \bar{Z}_{21} = \emptyset, \\ m \cap \bar{Z}_{21} & \text{if } m \cap \bar{Z}_{21} \neq \emptyset \text{ and } m \cap \bar{Z}_{12} = \emptyset, \\ m & \text{otherwise.} \end{cases} \tag{A.8}$$

Note that $\gamma^*(m) = (1/2, 1/2)$ for any message m such that $m \cap \bar{Z}_{12} \neq \emptyset$ and $m \cap \bar{Z}_{21} \neq \emptyset$.[53] We then check the optimality of the outlet's equilibrium strategy β^* given the voter's equilibrium strategy γ^*. If $z \notin \bar{Z}_{12} \cup \bar{Z}_{21}$, then the outlet has no incentive to deviate from $\beta^*(z)$ because it induces her preferred policy for certain. If $z \in \bar{Z}_{12} \cup \bar{Z}_{21}$, then the outlet also has no incentive to deviate because any available message induces her preferred policy with probability at most $1/2$. Finally, note that if $z \in Z(\alpha_1^*, \alpha_2^*) \backslash (Z_{02}(\alpha_1^*, \alpha_2^*) \cup Z_{01}(\alpha_1^*, \alpha_2^*))$, then $\gamma^*(\beta^*(z)) = y^v(z)$ holds because the outlet fully discloses the information. If $z \in Z_{02}(\alpha_1^*, \alpha_2^*) \cup Z_{01}(\alpha_1^*, \alpha_2^*)$, then $\gamma^*(\beta^*(z)) = (1/2, 1/2) = y^v(z)$ holds. Therefore, it is a desired PBE.

(ii) It is a corollary of Proposition 2-(i). Hence, the proof is omitted. ■

A.3 Proof of Claim 1

We show that the following is a PBE. For any $q \in (0, p)$:

$$\begin{aligned} \alpha_1^*(x_1) &= \begin{cases} q/p & \text{if } x_1 = 0, \\ 1 - q/p & \text{if } x_1 = r, \\ 0 & \text{otherwise;} \end{cases} \\ \alpha_2^*(x_2) &= \begin{cases} q & \text{if } x_2 = 0, \\ 1 - q & \text{if } x_2 = r, \\ 0 & \text{otherwise;} \end{cases} \\ \mathcal{P}^*(\tilde{z}|m) &= \begin{cases} 1 & \text{if } m = z \text{ and } \tilde{z} = z, \\ \frac{1}{2} & \text{if } m = \bar{Z}_{12} \cup \bar{Z}_{21} \text{ and } \tilde{z} = (0, r) \text{ or } (r, 0), \\ 0 & \text{otherwise;} \end{cases} \end{aligned} \tag{A.9}$$

[52] Because $z \in \bar{Z}_{12}$ and $z' \in \bar{Z}_{21}$, then $|x_1| \leq |x_2|$ and $|x_1'| \geq |x_2'|$ hold. That is, $a \in [0,1]$, and it is well-defined unless $|x_1| = |x_2|$ and $|x_1'| = |x_2'|$.

[53] Because $Z_{02}(\alpha_1^*, \alpha_2^*) \neq \emptyset$ and $Z_{01}(\alpha_1^*, \alpha_2^*) \neq \emptyset$, message $m = \bar{Z}_{12} \cup \bar{Z}_{21}$ is sent on the equilibrium path. Notice that any policy pair in regions $Z_{02} \cup Z_{01}$ is indifferent for the voter. Furthermore, because $Z_{12}(\alpha_1^*, \alpha_2^*) = Z_{21}(\alpha_1^*, \alpha_2^*) = \emptyset$, $\gamma^*(\bar{Z}_{12} \cup \bar{Z}_{21}) = (1/2, 1/2)$ holds.

The outlet's strategy β^* and the voter's belief for off-the-equilibrium-path messages are given by (A.6), (A.7) and (A.8), respectively. The optimality of strategies β^* and γ^* can be shown by the same argument used in the proof of Proposition 2. Hence, it remains to show the optimality of strategies α_1^* and α_2^*, and the consistency of belief $\mathcal{P}^*$. It is worthwhile to remark that $\gamma^*(\bar{Z}_{12} \cup \bar{Z}_{21}) = (1/2, 1/2)$.

First, consider candidate 1's behavior. Candidate 1's winning probabilities from strategies $\alpha_1 = 0$ and r are $U_1(0, \alpha_2^*, \beta^*, \gamma^*) = U_1(r, \alpha_2^*, \beta^*, \gamma^*) = 1 - p/2$, respectively. It is then sufficient to show that $U_1(\alpha_1', \alpha_2^*, \beta^*, \gamma^*) \leq 1 - p/2$ for any strategy $\alpha_1' \in \Delta(X)^*$. Candidate 1's winning probabilities given policies $x_2 = 0$, r and l are $\mu_1(x_1, 0) \leq 1/2$, $\mu_1(x_1, r) \leq 1/2$ and $\mu_1(x_1, l) \leq 1$ for any $x_1 \in X$ because possible policy pairs lie in regions $Z_0 \cup Z_{22} \cup Z_{21}$, $Z_0 \cup Z_{22} \cup \bar{Z}_{12} \cup Z_{21}$ and $Z_0 \cup Z_{11} \cup Z_{22} \cup \bar{Z}_{21}$, respectively. Hence, for any $\alpha_1' \in \Delta(X)^*$:

$$U_1(\alpha_1', \alpha_2^*, \beta^*, \gamma^*) \leq \frac{1}{2}pq + \frac{1}{2}p(1-q) + (1-p) = 1 - \frac{1}{2}p. \tag{A.10}$$

That is, α_1^* is optimal for candidate 1.

Next, consider candidate 2's behavior. Candidate 2's winning probabilities from strategies $\alpha_2 = 0$ and r are $U_2(\alpha_1^*, 0, \beta^*, \gamma^*) = U_2(\alpha_1^*, r, \beta^*, \gamma^*) = 1/2$, respectively. It is then sufficient to show that $U_2(\alpha_1^*, \alpha_2', \beta^*, \gamma^*) \leq 1/2$ for any strategy $\alpha_2' \in \Delta(X)^*$. Candidate 2's winning probabilities given policies $x_1 = 0$ and r are $\mu_2(0, x_2) \leq 1/2$ and $\mu_2(r, x_2) \leq 1/2$ for any $x_2 \in X$ because possible policy pairs lie in regions $Z_0 \cup Z_{11} \cup Z_{12}$ and $Z_0 \cup Z_{11} \cup Z_{12} \cup \bar{Z}_{21}$, respectively. Hence, for any $\alpha_2' \in \Delta(X)^*$:

$$U_2(\alpha_1^*, \alpha_2', \beta^*, \gamma^*) \leq \frac{1}{2}q + \frac{1}{2}(1-q) = \frac{1}{2}. \tag{A.11}$$

That is, α_2^* is optimal for candidate 2. Finally, it is straightforward that belief $\mathcal{P}^*$ is consistent with Bayes' rule. Thus, it is a PBE. ■

A.4 Proof of Theorem 1

A.4.1 Preliminary

Lemma 1 *In any equilibrium, $\gamma^*(\beta^*(z)) = y^v(z)$ for any $z \in Z_{11} \cup Z_{22}$.*

Proof of Lemma 1. Suppose, in contrast, that there exists an equilibrium $(\alpha_1^*, \alpha_2^*, \beta^*, \gamma^*; \mathcal{P}^*)$ such that $\gamma^*(\beta^*(z)) \neq y^v(z)$ for some policy pair $z \in Z_{11} \cup Z_{22}$. However, the outlet observing policy pair z also prefers action $y = y^v(z)$ to any other actions, and that preferred action can be induced by sending message $m = z$ because of Requirement 1. That is, the outlet has an incentive to deviate, which is a contradiction. ■

A.4.2 Proof of Theorem 1

(Necessity) Suppose, in contrast, that there exists the $(0,0)$ equilibrium when either (i) $b > r/2$ with $b \neq r$, or (ii) $b = r$ and $p > 1/2$ holds. Note that, in both scenarios, $Z_0(0,0) = \{(0,0)\}$, $Z_{11}(0,0) = \{(0,l),(r,l)\}$, $Z_{21}(0,0) = \{(r,0)\}$, and then $Z(0,0) = Z_0(0,0) \cup Z_{11}(0,0) \cup Z_{21}(0,0)$. First, suppose that $b > r/2$ with $b \neq r$ (Case (i)). By Requirement 2, there are the following three cases to be considered.

Case (i)-1: $\gamma^*(\beta^*(r,0)) = (1,0)$.

Note that $U_1(0,0,\beta^*,\gamma^*) = 1 - p/2$. If candidate 1 deviates to strategy $\alpha_1 = r$, then his winning probability is $U_1(r,0,\beta^*,\gamma^*) = 1$ by Lemma 1. That is, candidate 1 has an incentive to deviate to $\alpha_1 = r$, which is a contradiction.

Case (i)-2: $\gamma^*(\beta^*(r,0)) = (0,1)$.

To support this equilibrium, $\gamma^*(m) = (0,1)$ should hold for any message $m \in M(r,0)$; otherwise, the outlet observing policy pair $z = (r,0)$ deviates to such a message. Hence, $\gamma^*(\beta^*(z)) = (0,1)$ must hold for any policy pair $z \in Z_{12}$; otherwise, the outlet observing policy pairs lying in region Z_{12} has an incentive to deviate to a message containing policy pair $z = (r,0)$. Note that $U_2(0,0,\beta^*,\gamma^*) = 1-p/2$. If the media bias is $r/2 < b < r$, then consider candidate 2's deviation to $\alpha_2 = b$. The realized policy pair under this deviation is either $z = (0,b) \in Z_{12}$ or $(r,b) \in Z_{22}$. Hence, by Lemma 1, candidate 2's winning probability from strategy $\alpha_2 = b$ is $U_2(0,b,\beta^*,\gamma^*) = 1$. That is, candidate 2 has an incentive to deviate, which is a contradiction. If the media bias is $b > r$, then we can derive the similar contradiction by considering candidate 2's deviation to $\alpha_2 = x_2' \in (r,b)$.

Case (i)-3: $\gamma^*(\beta^*(r,0)) = (1/2,1/2)$.

To support this equilibrium, the outlet observing policy pair $z = (r,0)$ must be pooling with the outlet observing policy pair either $z = (0,l)$ or (r,l); otherwise, $\gamma^*(\beta^*(r,0)) = (0,1)$ holds. That is, either $\gamma^*(\beta^*(0,l)) = (1/2,1/2)$ or $\gamma^*(\beta^*(r,l)) = (1/2,1/2)$ holds. However, by Lemma 1, this is impossible, which is a contradiction.

Next, we suppose that $b = r$ and $p > 1/2$ (Case (ii)). Likewise, there are the three cases to be checked dependent on the voter's response to message $m = \beta^*(r,0)$. If either $\gamma^*(\beta^*(r,0)) = (1,0)$ or $(1/2,1/2)$, then we can derive contradictions by the same argument used in Case (i)-1 and (i)-3.

Hence, it remains to check the scenario where $\gamma^*(\beta^*(r,0)) = (0,1)$. To support this equilibrium, $\gamma^*(m) = (0,1)$ should hold for any message $m \in M(r,0)$; otherwise, the outlet observing policy pair $z = (r,0)$ deviates. Hence, it implies that $\gamma^*(\beta^*(z)) = (0,1)$ should hold for any policy pair $z \in \bar{Z}_{12}$; otherwise, the outlet deviates to send a message containing $(r,0)$. Now, candidate 2's winning probabilities from strategies $\alpha_2 = 0$ and r are $U_2(0,0,\beta^*,\gamma^*) = 1 - p/2$ and $U_2(0,r,\beta^*,\gamma^*) = (1+p)/2$, respectively. However, because $p > 1/2$, $U_2(0,r,\beta^*,\gamma^*) > U_2(0,0,\beta^*,\gamma^*)$. That is, candidate 2 has an incentive to deviate, which is a contradiction.

(Sufficiency) Suppose that $b \leq r/2$. Note that $Z_0(0,0) = \{(0,0)\}$, $Z_{11}(0,0) = \{(0,l),(r,l)\}$, $Z_{22} = \{(r,0)\}$ and $Z(0,0) = Z_0(0,0) \cup Z_{11}(0,0) \cup Z_{22}(0,0)$. We then show that there exists a $(0,0)$ equilibrium supported by strategy β^* and off-the-equilibrium-path beliefs $\mathcal{P}^*$ given by (A.6), (A.7) and (A.8), respectively. First, it is obvious that γ^* is optimal given belief $\mathcal{P}^*$. It is worthwhile to remark that $\gamma^*(m) = (1/2,1/2)$ holds for any message m such that $m \cap \bar{Z}_{12} \neq \emptyset$ and $m \cap \bar{Z}_{21} \neq \emptyset$. Second, we can show that the optimality of the outlet's strategy β^* by the similar argument used in the proof of Proposition 2. Third, we show the optimality of the candidates' strategies α_1^* and α_2^* given the others' strategies. Note that candidate 1's winning probability from strategy $\alpha_1 = 0$ is $U_1(0,0,\beta^*,\gamma^*) = 1-p/2$. It is then sufficient to show that $U_1(\alpha_1',0,\beta^*,\gamma^*) \leq 1-p/2$ for any strategy $\alpha_1' \in \Delta(X)^*$. Candidate 1's winning probabilities given policies $x_2 = 0$ and l are $\mu_1(x_1,0) \leq 1/2$ and $\mu_1(x_1,l) \leq 1$ for any $x_1 \in X$ because possible policy pairs lie in regions $Z_0 \cup Z_{22} \cup Z_{21}$ and $Z_0 \cup Z_{11} \cup Z_{22} \cup \bar{Z}_{21}$, respectively. Hence, for any $\alpha_1' \in \Delta(X)^*$:

$$U_1(\alpha_1',0,\beta^*,\gamma^*) \leq \frac{1}{2}p + (1-p) = 1 - \frac{1}{2}p. \tag{A.12}$$

That is, candidate 1 has no incentive to deviate. Likewise, note that candidate 2's winning probability from strategy $\alpha_2 = 0$ is $U_2(0,0,\beta^*,\gamma^*) = 1 - p/2$. It is then sufficient to show that $U_2(0,\alpha_2',\beta^*,\gamma^*) \leq 1 - p/2$ for any strategy $\alpha_2' \in \Delta(X)^*$. Candidate 2's winning probabilities given policies $x_1 = 0$ and r are $\mu_2(0,x_2) \leq 1/2$ and $\mu_2(r,x_2) \leq 1$ because possible policy pairs lie in regions $Z_0 \cup Z_{11} \cup Z_{12}$ and $Z_0 \cup Z_{11} \cup Z_{22} \cup \bar{Z}_{21}$, respectively. Hence, for any $\alpha_2' \in \Delta(X)^*$:

$$U_2(0,\alpha_2',\beta^*,\gamma^*) \leq \frac{1}{2}p + (1-p) = 1 - \frac{1}{2}p. \tag{A.13}$$

That is, candidate 2 has no incentive to deviate. Finally, because $\bar{Z}_{12}(0,0) = \bar{Z}_{21}(0,0) = \emptyset$, only the fully disclosure messages are used on the equilibrium path. That is, $\mathcal{P}^*$ is consistent with Bayes' rule. Thus, it is a PBE.

Next, suppose that $b = r$ and $p \leq 1/2$. We show that the following is a PBE: $\alpha_1^* = \alpha_2^* = 0$;

$$\beta^*(z) = \begin{cases} z & \text{if } z \in Z_0 \cup Z_{11} \cup Z_{22} \cup \{(r,0)\}, \\ \{z, (r,0)\} & \text{if } z \in \bar{Z}_{12}, \\ \{z, z'\} & \text{if } z \in \bar{Z}_{21} \backslash \{(r,0)\} \text{ where } z = (x_1, x_2) \text{ and } z' = (x_2, x_1) \in \bar{Z}_{12}; \end{cases} \tag{A.14}$$

for off-the-equilibrium-path message $m' \equiv \{z, z'\}$ with $z = (x_1, x_2) \in \bar{Z}_{21}$ and $z' = (x_2, x_1) \in \bar{Z}_{12}$:

$$\mathcal{P}^*(\tilde{z}|m') = \begin{cases} 1/2 & \text{if } \tilde{z} = z \text{ or } z', \\ 0 & \text{otherwise;} \end{cases} \tag{A.15}$$

for other off-the-equilibrium-path messages:

$$S(\mathcal{P}^*(\cdot|m)) \subseteq \begin{cases} \{(r,0)\} & \text{if } m \in M(r,0), \\ m \cap \bar{Z}_{21} & \text{if } m \notin M(r,0), m \neq m', \text{ and } m \cap \bar{Z}_{21} \neq \emptyset, \\ m & \text{otherwise.} \end{cases} \tag{A.16}$$

Note that $\gamma^*(m) = (0,1)$ for any message $m \in M(r,0)$, and $\gamma^*(m') = (1/2, 1/2)$.

First, we show that the outlet never deviates from strategy β^*. It is obvious that the outlet observing a policy pair in region $Z_0 \cup Z_{11} \cup Z_{22} \cup \bar{Z}_{12}$ has no incentive to deviate because following β^* induces her more preferred policy for certain. For the outlet observing a policy pair $z \in \bar{Z}_{21} \backslash \{(r,0)\}$, any available message cannot induce her preferred policy with probability more than 1/2. It is obvious that the outlet observing policy pair $z = (r,0)$ has no incentive to deviate because any message $m \in M(r,0)$ induces $\gamma^*(m) = (0,1)$. Thus, the outlet has no incentive to deviate from β^*.

Second, we show that candidate 1 never deviates from strategy α_1^*. Note that candidate 1's winning probability from strategy $\alpha_1 = 0$ is $U_1(0, 0, \beta^*, \gamma^*) = 1 - p/2$. It is then sufficient to show that $U_1(\alpha_1', 0, \beta^*, \gamma^*) \leq 1 - p/2$ for any strategy $\alpha_1' \in \Delta(X)^*$. Candidate 1's winning probabilities given policies $x_2 = 0$ and l are $\mu_1(x_1, 0) \leq 1/2$ and $\mu_1(x_1, l) \leq 1$ for any $x_1 \in X$ because policy pair lie in regions $Z_0 \cup Z_{22} \cup Z_{21}$ and $Z_0 \cup Z_{11} \cup Z_{22} \cup \bar{Z}_{21}$, respectively. Hence, for any $\alpha_1' \in \Delta(X)^*$:

$$U_1(\alpha_1', 0, \beta^*, \gamma^*) \leq \frac{1}{2}p + (1-p) = 1 - \frac{1}{2}p. \tag{A.17}$$

That is, candidate 1 has no incentive to deviate.

Third, we show that candidate 2 never deviates from strategy α_2^*. Note that candidate 2's winning probability from strategy $\alpha_2 = 0$ is $U_2(0, 0, \beta^*, \gamma^*) = 1 - p/2$. For any policy $x_2 \in X$, possible policy pairs lie in regions $Z_0 \cup Z_{11} \cup Z_{12}$ if $x_1 = 0$, and regions $Z_0 \cup Z_{11} \cup \bar{Z}_{21}$ if $x_1 = r$.

Hence, candidate 2's winning probabilities are determined as follows:

$$\mu_2(0, x_2) = \begin{cases} 0 & \text{if } x_2 \in [-\bar{x}, 0) \cup [2b, \bar{x}], \\ 1/2 & \text{if } x_2 = 0, \\ 1 & \text{if } x_2 \in (0, 2b); \end{cases} \tag{A.18}$$

$$\mu_2(r, x_2) = \begin{cases} 0 & \text{if } x_2 \in [-\bar{x}, -r) \cup (r, \bar{x}], \\ 1/2 & \text{if } x_2 \in [-r, r] \backslash \{0\}, \\ 1 & \text{if } x_2 = 0. \end{cases} \tag{A.19}$$

That is, the maximum winning probability is either $1-p/2$ induced by strategy $\alpha_2 = 0$ or $(1+p)/2$ induced by strategy $\alpha_2 \in (0, r]$. Because $p \leq 1/2$, candidate 2 has no incentive to deviate from α_2^*. Finally, it is obvious that belief $\mathcal{P}^*$ is consistent with Bayes' rule. Thus, it is a PBE. ■

A.5 Proof of Theorem 2

A.5.1 Preliminary (i): Key properties of USEs.

First, we introduce the following notation. For the outlet's simple strategy $\bar{\beta} \in B$ and the voter's strategy $\gamma \in \Gamma$, where $\gamma(\bar{Z}_{12} \cup \bar{Z}_{21}) = (c, 1-c)$ with $c \in \{0, 1/2, 1\}$, define $\bar{\gamma}_c : Z \to \Delta(Y)$ by $\bar{\gamma}_c(z) \equiv \gamma(\bar{\beta}(z))$ for any $z \in Z$, which we call the *induced outcome of the news-reporting stage*. Define:

$$\bar{\Gamma} \equiv \left\{ \bar{\gamma}_c \in \Delta(Y)^Z \;\middle|\; \begin{array}{l} \text{there exist } \bar{\beta} \in B \text{ and } \gamma \in \Gamma \text{ such that} \\ \text{(i) } \bar{\gamma}_c = \gamma \circ \bar{\beta}, \\ \text{(ii) } \bar{\beta} \text{ is a best response to } \gamma, \\ \text{(iii) if } m = z = (x_1, x_2) \text{ and } x_1 = x_2, \text{ then } \gamma(z) = (1/2, 1/2) \end{array} \right\}, \tag{A.20}$$

which is, roughly, the set of induced outcomes of the news-reporting stage that can be supported in some PBE constructed by the simple strategies.[54] Let $\bar{U}_i : \Delta(X)^{*2} \times \bar{\Gamma} \to \mathbb{R}$ be opportunistic-type candidate i's expected utility when the induced outcome in the news-reporting stage is $\bar{\gamma}_c$, which is defined by:

$$\bar{U}_i(\alpha_1, \alpha_2, \bar{\gamma}_c) \equiv \sum_{z \in Z} \Pr(y_i | \bar{\gamma}_c(z)) \Pr(z | \alpha_1, \alpha_2, \theta_i = O). \tag{A.21}$$

[54]Condition (iii) is a tie-breaking rule for the voter. Without this condition, the equilibrium selection discussed above does not work well.

Define $Z_{00} \equiv \{z \in Z | v(z, y_1) = v(z, y_2) \text{ and } w(z, y_1) = w(z, y_2)\}$. We then show the following lemmas, which are useful properties of USEs.

Lemma 2

(i) Suppose that $b \leq r/2$. Then, there exists no USE with the front-runner.

(ii) Suppose that $b > r/2$. If there exists a USE with the front-runner, then it is candidate 1.[55]

Proof. (i) Suppose, in contrast, that there exists USE $(\alpha_1^*, \alpha_2^*, \beta^*, \gamma^*; \mathcal{P}^*)$ such that the induced outcome of the news-reporting stage is $\bar{\gamma}_1$ when $b < r/2$. Hence:

$$\bar{U}_1(\alpha_1^*, \alpha_2^*, \bar{\gamma}_1) = 1 - \frac{1}{2} \sum_{z \in Z_{00}} \Pr(z | \alpha_1^*, \alpha_2^*, \theta_1 = O), \tag{A.22}$$

$$\bar{U}_2(\alpha_1^*, \alpha_2^*, \bar{\gamma}_1) = \frac{1}{2} \sum_{z \in Z_{00}} \Pr(z | \alpha_1^*, \alpha_2^*, \theta_2 = O) + (1 - p). \tag{A.23}$$

To hold this equilibrium, $\sum_{z \in Z_{00}} \Pr(z | \alpha_1^*, \alpha_2^*, \theta_1 = O) = 0$ should hold; otherwise, candidate 1 deviates to strategy α_1' such that $S(\alpha_1') \cap S(\alpha_2^*) = \emptyset$. That is, $\sum_{z \in Z_{00}} \Pr(z | \alpha_1^*, \alpha_2^*, \theta_1 = O, \theta_2 = O) = 0$. Furthermore, because $b \leq r/2$, $\sum_{z \in Z_{00}} \Pr(z | \alpha_1^*, \alpha_2^*, \theta_1 = I, \theta_2 = O) = 0$. Thus, $\bar{U}_2(\alpha_1^*, \alpha_2^*, \bar{\gamma}_1) = 1 - p$. However, if candidate 2 deviates to strategy $\alpha_2 = \alpha_1^*$, then $\bar{U}_2(\alpha_1^*, \alpha_1^*, \bar{\gamma}_1) > \bar{U}_2(\alpha_1^*, \alpha_2^*, \bar{\gamma}_1)$ holds, which is a contradiction. Likewise, we can derive a contradiction when the induced outcome of the news-reporting stage is $\bar{\gamma}_0$. In other words, there exists no front-runner in this scenario.

(ii) Suppose, in contrast, that there exists UCE $(\alpha_1^*, \alpha_2^*, \beta^*, \gamma^*; \mathcal{P}^*)$ where the induced outcome of the news-reporting stage is $\bar{\gamma}_0$ when $b > r/2$. First, we suppose that $r/2 < b < |l|$. For any $\alpha_1, \alpha_2 \in \Delta([0, b])^*$:

$$\bar{U}_1(\alpha_1, \alpha_2^*, \bar{\gamma}_0) = \frac{1}{2} \sum_{z \in Z_{00}} \Pr(z | \alpha_1, \alpha_2^*, \theta_1 = O) + (1 - p). \tag{A.24}$$

$$\bar{U}_2(\alpha_1^*, \alpha_2, \bar{\gamma}_0) = 1 - \frac{1}{2} \sum_{z \in Z_{00}} \Pr(z | \alpha_1^*, \alpha_2, \theta_2 = O). \tag{A.25}$$

Because α_1^* is an equilibrium strategy, $\bar{U}_1(\alpha_1^*, \alpha_2^*, \bar{\gamma}_0) \geq \bar{U}_1(\alpha_1, \alpha_2^*, \bar{\gamma}_0)$ holds for any $\alpha_1 \in \Delta([0, b])^*$. That is, $\sum_{z \in Z_{00}} \Pr(z | \alpha_1^*, \alpha_2^*, \theta_1 = O) > 0$; otherwise, candidate 1 has an incentive to deviate to $\alpha_1 = \alpha_2^*$. Thus, $\sum_{z \in Z_{00}} \Pr(z | \alpha_1^*, \alpha_2^*) > 0$ holds. Note that:

$$\sum_{z \in Z_{00}} \Pr(z | \alpha_1^*, \alpha_2^*) = p \sum_{z \in Z_{00}} \Pr(z | \alpha_1^*, \alpha_2^*, \theta_2 = O) + (1 - p) \sum_{z \in Z_{00}} \Pr(z | \alpha_1^*, \alpha_2^*, \theta_2 = I). \tag{A.26}$$

[55] This property depends on the assumption that $|r| < |l|$. Otherwise, candidate 2 can be the front-runner.

Because $\alpha_1^* \in \Delta([0,b])^*$, $\sum_{z\in Z_{00}} \Pr(z|\alpha_1^*, \alpha_2^*, \theta_2 = O) > 0$ should hold. In other words, either $S(\alpha_1^*) \cap S(\alpha_2^*) \neq \emptyset$ or $r \in S(\alpha_2^*)$ holds. Now, suppose that candidate 2 deviates to strategy α_2' such that $(S(\alpha_1^*) \cup \{r\}) \cap S(\alpha_2') = \emptyset$.[56] By Construction, $\sum_{z\in Z_{00}} \Pr(z|\alpha_1^*, \alpha_2', \theta_2 = O) = 0$. Hence, $\bar{U}_2(\alpha_1^*, \alpha_2', \bar{\gamma}_0) = 1 > \bar{U}_2(\alpha_1^*, \alpha_2^*, \bar{\gamma}_0)$. This means that candidate 2 has an incentive to deviate, which is a contradiction.

Next, we suppose that $b \geq |l|$. For any $\alpha_2 \in \Delta([0,b])^*$, $\bar{U}_2(\alpha_1^*, \alpha_2, \bar{\gamma}_0)$ is given by (A.25). Because $\bar{U}_2(\alpha_1^*, \alpha_2^*, \bar{\gamma}_0) \geq \bar{U}_2(\alpha_1^*, \alpha_2, \bar{\gamma}_0)$ must hold for any $\alpha_2 \in \Delta([0,b])^*$, $\sum_{z\in Z_{00}} \Pr(z|\alpha_1^*, \alpha_2^*, \theta_2 = O) = 0$ should hold; otherwise, candidate 2 deviates to strategy α_2' such that $(S(\alpha_1^*) \cup \{r\}) \cap S(\alpha_2') = \emptyset$. Hence, $\sum_{z\in Z_{00}} \Pr(z|\alpha_1^*, \alpha_2^*, \theta_1 = O, \theta_2 = O) = 0$ holds. That is:

$$\begin{aligned}\bar{U}_1(\alpha_1^*, \alpha_2^*, \bar{\gamma}_0) &= \frac{p}{2} \sum_{z\in Z_{00}} \Pr(z|\alpha_1^*, \alpha_2^*, \theta_1 = O, \theta_2 = O) + \frac{1-p}{2}\alpha_1^*(|l|) + (1-p) \sum_{x_1 \in [0,|l|)} \alpha_1^*(x_1) \\ &= \frac{1-p}{2}\alpha_1^*(|l|) + (1-p) \sum_{x_1 \in [0,|l|)} \alpha_1(x_1)^*. \end{aligned} \tag{A.27}$$

Because $\bar{U}_1(\alpha_1^*, \alpha_2^*, \bar{\gamma}_0) \geq \bar{U}_1(\alpha_1, \alpha_2^*, \bar{\gamma}_0)$ holds for any $\alpha_1 \in \Delta([0,b])^*$, $S(\alpha_1^*) \subseteq [0,|l|)$ must hold. In other words, $\bar{U}_1(\alpha_1^*, \alpha_2^*, \bar{\gamma}_0) = 1-p$. Now, suppose, in contrast, that there exists $x_2 \in S(\alpha_2^*) \cap [0,|l|)$. Because $\sum_{z\in Z_{00}} \Pr(z|\alpha_1^*, \alpha_2^*, \theta_1 = O, \theta_2 = O) = 0$, $x_2 \notin S(\alpha_1^*)$. If candidate 1 deviates to strategy $\alpha_1 = x_2$ in this scenario, then his expected utility after this deviation is $\bar{U}_1(x_2, \alpha_2^*, \bar{\gamma}) = p\alpha_2^*(x_2)/2 + 1-p > 1-p = \bar{U}_1(\alpha_1^*, \alpha_2^*, \bar{\gamma}_0)$, which is a contradiction. Thus, we can say that $S(\alpha_2^*) \subseteq [|l|, b]$ must hold. However, given such α_1^* and α_2^*, $\bar{Z}_{12}(\alpha_1^*, \alpha_2^*) \neq \emptyset$ and $\bar{Z}_{21}(\alpha_1^*, \alpha_2^*) = \emptyset$. Hence, $\gamma^*(\bar{Z}_{12} \cup \bar{Z}_{21}) = (1,0)$ is the voter's best response supported by the consistent belief. That is, the induced outcome in the news-reporting stage should be $\bar{\gamma}_1$, which is a contradiction. Therefore, if there exists a USE with the front-runner, then the front-runner should be candidate 1. ■

Lemma 3 *Suppose that $r/2 < b < r$. Then, there exists no USE such that $S(\alpha_2^*) \cap [0, -r+2b) \neq \emptyset$.*

Proof. Suppose, in contrast, that there exists USE $(\alpha_1^*, \alpha_2^*, \beta^*, \gamma^*; \mathcal{P}^*)$ such that $S(\alpha_2^*) \cap [0, -r+2b) \neq \emptyset$ when $2/r < b < r$. By Lemma 2-(ii), the induced outcome of the news-reporting stage is either $\bar{\gamma}_1$ or $\bar{\gamma}_{1/2}$. First, we suppose that the induced outcome of the news-reporting stage is $\bar{\gamma}_1$. Hence:

$$\bar{U}_1(\alpha_1^*, \alpha_2^*, \bar{\gamma}_1) = 1 - \frac{1}{2} \sum_{z\in Z_{00}} \Pr(z|\alpha_1^*, \alpha_2^*, \theta_1 = O). \tag{A.28}$$

Now, we show that $\sum_{z\in Z_{00}} \Pr(z|\alpha_1^*, \alpha_2^*, \theta_1 = O) > 0$. Suppose, in contrast, that $\sum_{z\in Z_{00}} \Pr(z|\alpha_1^*, \alpha_2^*, \theta_1 =$

[56] Because we restrict our attention to the scenario where any mixed strategies have finite supports, such α_2' always exists.

$O) = 0$. Hence, by the hypothesis and $(x_1, x_2) \notin Z_{22}$ for any $x_i \in S(\alpha_i^*)$:

$$\begin{aligned}
\bar{U}_2(\alpha_1^*, \alpha_2^*, \bar{\gamma}_1) &= \frac{1}{2} \sum_{z \in Z_{00}} \Pr(z|\alpha_1^*, \alpha_2^*, \theta_2 = O) + \sum_{z \in Z_{22}} \Pr(z|\alpha_1^*, \alpha_2^*, \theta_2 = O) \\
&= (1-p) \left(\frac{1}{2} \sum_{z \in Z_{00}} \Pr(z|\alpha_1^*, \alpha_2^*, \theta_1 = I, \theta_2 = O) + \sum_{z \in Z_{22}} \Pr(z|\alpha_1^*, \alpha_2^*, \theta_1 = I, \theta_2 = O) \right) \\
&= (1-p) \sum_{x_2 \in [-r+2b, b]} \alpha_2^*(x_2). \qquad \text{(A.29)}
\end{aligned}$$

Because $\bar{U}_2(\alpha_1^*, \alpha_2^*, \bar{\gamma}_1) \geq \bar{U}_2(\alpha_1^*, \alpha_2, \bar{\gamma}_1)$ for any $\alpha_2 \in \Delta([0, b])^*$, $S(\alpha_2^*) \subseteq [-r + 2b, b]$ should hold, which is a contradiction to $S(\alpha_2^*) \cap [0, -r + 2b) \neq \emptyset$. Thus, $\sum_{z \in Z_{00}} \Pr(z|\alpha_1^*, \alpha_2^*, \theta_1 = O) > 0$ holds. That is, $\bar{U}_1(\alpha_1^*, \alpha_2^*, \bar{\gamma}_c) < 1$. However, if candidate 1 deviates to strategy α_1' such that $S(\alpha_1') \cap S(\alpha_2^*) = \emptyset$, then $\bar{U}_1(\alpha_1', \alpha_2^*, \bar{\gamma}_1) = 1$, which is a contradiction.

Next, we suppose that the induced outcome of the news-reporting stage is $\bar{\gamma}_{1/2}$. Hence:

$$\bar{U}_2(\alpha_1^*, \alpha_2^*, \bar{\gamma}_{1/2}) = \frac{1}{2} \left(p + (1-p) \sum_{x_2 \in [0, -r+2b)} \alpha_2^*(x_2) \right) + (1-p) \sum_{x_2 \in [-r+2b, b]} \alpha_2^*(x_2). \qquad \text{(A.30)}$$

By the hypothesis, $\sum_{x_2 \in [0, -r+2b)} \alpha_2^*(x_2) > 0$ holds. Now, if candidate 2 deviates to strategy $\alpha_2 = -r + 2b$, then $\bar{U}_2(\alpha_1^*, -r + 2b, \bar{\gamma}_{1/2}) = p/2 + (1-p) > \bar{U}_2(\alpha_1^*, \alpha_2^*, \bar{\gamma}_{1/2})$. Therefore, candidate 2 has an incentive to deviate, which is a contradiction. ■

Lemma 4 *Suppose that $b \geq |l|$. If there exists a USE where $S(\alpha_1^*) \cap [|l|, b] \neq \emptyset$, then the induced outcome in the news-reporting stage is $\bar{\gamma}_1$.*

Proof. Suppose, in contrast, that there exists a USE in which $S(\alpha_1^*) \cap [|l|, b] \neq \emptyset$ and the induced outcome of the news-reporting stage is not $\bar{\gamma}_1$. By Lemma 2-(ii), the induced outcome of the news-reporting stage is $\bar{\gamma}_{1/2}$. Hence:

$$\bar{U}_1(\alpha_1^*, \alpha_2^*, \bar{\gamma}_{1/2}) = \frac{1}{2} p + (1-p) \left(\sum_{x_1 \in [0, |l|)} \alpha_1^*(x_1) + \frac{1}{2} \sum_{x_1 \in [|l|, b]} \alpha_1^*(x_1) \right). \qquad \text{(A.31)}$$

Because $S(\alpha_1^*) \cap [|l|, b] \neq \emptyset$, $\sum_{x_1 \in [|l|, b]} \alpha_1^*(x_1) > 0$ holds. However, if candidate 1 deviates to strategy

α_1' defined by:

$$\alpha_1'(x_1) \equiv \begin{cases} \alpha_1^*(0) + \sum_{x_1 \in [|l|, b]} \alpha_1^*(x_1) & \text{if } x_1 = 0, \\ \alpha_1^*(x_1) & \text{if } x_1 \in (0, |l|), \\ 0 & \text{otherwise.} \end{cases} \tag{A.32}$$

Then, $\bar{U}_1(\alpha_1', \alpha_2, \bar{\gamma}_{1/2}) = 1 - p/2 > \bar{U}_1(\alpha_1^*, \alpha_2^*, \bar{\gamma}_{1/2})$. Thus, candidate 1 has an incentive to deviate, which is a contradiction. ■

A.5.2 Preliminary (ii): Construction of USEs

Next, we provide the lemmas that tell us how to construct the desired USEs.

Lemma 5 *Suppose that $0 < b \leq r/2$. Then, for any $\alpha \in \Delta([0, b])$, there exists a UCE such that $\alpha_1^* = \alpha_2^* = \alpha$.*

Proof. Fix $\alpha \in \Delta([0, b])^*$, arbitrarily. We show that the following is a USE: $\alpha_1^* = \alpha_2^* = \alpha$, and the outlet's strategy β^* and the voter's belief $\mathcal{P}^*$ for off-the-equilibrium-path messages are given by (A.6), (A.7) and (A.8), respectively. It is obvious that $\beta^* \in B$ and $\gamma^* \in \Gamma$. If α is a degenerate distribution, then $\bar{Z}_{12}(\alpha_1^*, \alpha_2^*) \cup \bar{Z}_{21}(\alpha_1^*, \alpha_2^*) = \emptyset$. Hence, we can show the optimality of β^* and γ^* by the similar argument used in the proof of Proposition 2-(i). If α is a nondegenerate distribution, then $Z_{12}(\alpha_1^*, \alpha_2^*) \neq \emptyset$ and $Z_{21}(\alpha_1^*, \alpha_2^*) \neq \emptyset$. Hence, the consistent belief implies that $\gamma^*(\bar{Z}_{12} \cup \bar{Z}_{21}) = (1/2, 1/2)$ because of the symmetry between strategies α_1^*, and α_2^*. In this scenario, it is also obvious that β^* is the outlet's best response because the outlet who observes policy pairs in the disagreement regions cannot induce his preferred outcome with probability more than 1/2. Thus, we can say that, in both scenarios, $\gamma^*(\beta^*(z)) = (1/2, 1/2)$ holds for any $z \in [0, b]^2$. Given β^*, γ^* and α_j^*, any pure strategy $\alpha_i \in [0, b]$ is indifferent for each candidate, which means that strategy α_i^* is a best response. Therefore, it is a USE. ■

Lemma 6 *Suppose that $r/2 < b < r$.*

(i) For any $\alpha \in \Delta([-r + 2b, b])^*$, there exists a USE where $\alpha_1^* = \alpha_2^* = \alpha$.

(ii) There exists a USE such that (1) $S(\alpha_1^*) = \{\varepsilon_1, \cdots, \varepsilon_N\}$ with $\alpha_1(x_1) = 1/N$ for any $x_1 \in S(\alpha_1^*)$ and $\varepsilon_i \in [0, -r + 2b)$ for any i; and (2) $\alpha_2^* \in \Delta([-r + 2b, b])^*$, where N satisfies $2(N-1)/(1 + 2(N-1)) < p \leq 2N/(1 + 2N)$.

Proof. (i) Fix $\alpha \in \Delta([-r+2b,b])^*$, arbitrarily. We show that the following is a USE: $\alpha_1^* = \alpha_2^* = \alpha$, and the outlet's strategy β^* and the voter's belief $\mathcal{P}^*$ for off-the-equilibrium-path messages are given by (A.6), (A.7) and (A.8), respectively. The same argument as in the proof of Lemma 5, we can show the optimality of β^* and γ^* such that $\gamma^*(\bar{Z}_{12} \cup \bar{Z}_{21}) = (1/2, 1/2)$. Then, it is sufficient to show the optimality of α_i^*. For candidate 1, $\bar{U}_1(\alpha, \alpha, \bar{\gamma}_{1/2}) = 1-p/2$. Notice that $\mu_1(x_1, x_2) = 1/2$ for any $x_1 \in [0,b]$ and $x_2 \in [-r+2b, b]$ because possible policy pairs lie in region $Z_{00} \cup Z_{12} \cup Z_{21}$. Likewise, $\mu_1(x_1, l) = 1$ for any $x_1 \in [0,b]$ because $(x_1, l) \in Z_{11}$. Thus, candidate 1 has no incentive to deviate because $\bar{U}_1(\alpha_1, \alpha, \bar{\gamma}_{1/2}) = 1-p/2$ for any $\alpha_1 \in \Delta([0,b])^*$. For candidate 2, $\bar{U}_2(\alpha, \alpha, \bar{\gamma}_{1/2}) = 1-p/2$. Notice that $\mu_2(x_1, x_2) = 1/2$ for any $x_1 \in [-r+2b, b]$ and $x_2 \in [0,b]$ because possible policy pairs lie in region $Z_{00} \cup Z_{12} \cup Z_{21}$. Furthermore, $\mu_2(r, x_2) \leq 1$ for any $x_2 \in [0,b]$ because possible policy pairs lie in region $Z_{21} \cup Z_{22}$. Thus, candidate 2 also never deviates because for any $\alpha_2 \in \Delta([0,b])^*$, $\bar{U}_2(\alpha, \alpha_2, \bar{\gamma}_{1/2}) \leq 1-p/2$. Therefore, it is a USE.

(ii) We show that the following is a USE: α_1^* and α_2^* satisfy condition (1) and (2); the outlet's strategy β^* is given by:

$$\beta^* = \begin{cases} \bar{Z}_{12} \cup \bar{Z}_{21} & \text{if } z \in Z_{12} \cup \bar{Z}_{21}, \\ z & \text{otherwise;} \end{cases} \tag{A.33}$$

the voter's belief for off-the equilibrium-path messages is given by:

$$S(\mathcal{P}^*(\cdot|m)) \subseteq \begin{cases} m \cap Z_{12} & \text{if } m \cap Z_{12} \neq \emptyset, \\ m \cap Z_{02} & \text{if } m \cap Z_{12} = \emptyset \text{ and } m \cap Z_{02} \neq \emptyset, \\ m & \text{otherwise.} \end{cases} \tag{A.34}$$

It is obvious that $\beta^* \in B$ and $\gamma^* \in \Gamma$. Note that because $\bar{Z}_{12}(\alpha_1^*, \alpha_2^*) \neq \emptyset$ and $\bar{Z}_{21}(\alpha_1^*, \alpha_2^*) = \emptyset$, the voter's consistent belief induces that $\gamma^*(\bar{Z}_{12} \cup \bar{Z}_{21}) = (1, 0)$, and then the induced outcome in the news-reporting stage is $\bar{\gamma}_1$. Hence, it is straightforward that β^* is the outlet's best response.

It remains to show the optimality of α_i^*. Because $S(\alpha_2^*) \subseteq [-r+2b, b]$, $\sum_{z \in Z_{00}} \Pr(z|\alpha_1^*, \alpha_2^*, \theta_1 = O) = 0$. Thus, $\bar{U}_1(\alpha_1^*, \alpha_2^*, \bar{\gamma}_1) = 1$; that is, candidate 1 has no incentive to deviate from α_1^*. Because $b \neq r$ and $S(\alpha_2^*) \subseteq [-r+2b, b]$, $\bar{U}_2(\alpha_1^*, \alpha_2^*, \bar{\gamma}_1) = 1-p$. Suppose that candidate 2 deviates to α_2' such that $S(\alpha_1^*) \cap S(\alpha_2') \neq \emptyset$. Hence:

$$\bar{U}_2(\alpha_1^*, \alpha_2', \bar{\gamma}_1) = \frac{1}{2} \sum_{z \in Z_{00}} \Pr(z|\alpha_1^*, \alpha_2'', \theta_2 = O) + (1-p)(1-q), \tag{A.35}$$

where $q \equiv \sum_{x_2 \in [0, -r+2b)} \alpha_2'(x_2)$. Now, consider the following strategy α_2'' given by:

$$\alpha_2''(x_2) \equiv \begin{cases} q & \text{if } x_2 = \varepsilon_1, \\ 0 & \text{if } x_2 \in (0, -r+2b), \\ \alpha_2'(x_2) & \text{otherwise.} \end{cases} \tag{A.36}$$

Then:

$$\bar{U}_2(\alpha_1^*, \alpha_2'', \bar{\gamma}_1) = \frac{pq}{2N} + (1-p)(1-q). \tag{A.37}$$

By construction of α_1^* and α_2'', $\bar{U}_2(\alpha_1^*, \alpha_2'', \bar{\gamma}_1) \geq \bar{U}_2(\alpha_1^*, \alpha_2', \bar{\gamma}_1)$ should hold. Furthermore, note that:

$$\begin{aligned} \bar{U}_2(\alpha_1^*, \alpha_2^*, \bar{\gamma}_1) - \bar{U}_2(\alpha_1^*, \alpha_2'', \bar{\gamma}_1) &= (1-p) - \frac{pq}{2N} - (1-p)(1-q) \\ &= q\left(1 - \frac{2N+1}{2N}p\right) \geq 0. \end{aligned} \tag{A.38}$$

That is, $\bar{U}_2(\alpha_1^*, \alpha_2^*, \bar{\gamma}_1) \geq \bar{U}_2(\alpha_1^*, \alpha_2', \bar{\gamma}_1)$ holds, which means that candidate 2 has no incentive to deviate to such strategy α_2'. If candidate 2 deviate to α_2' such that $S(\alpha_1^*) \cap S(\alpha_2') = \emptyset$ with $\alpha_2' \neq \alpha_2^*$, then $\bar{U}_2(\alpha_1^*, \alpha_2', \bar{\gamma}_1) \leq 1-p$. As a result, candidate 2 has no incentive to deviate from α_2^*. Therefore, it is a USE. ■

Lemma 7 *Suppose that $b \geq r$. Then, there exist USEs where $\alpha_1^* = x_1$ and $\alpha_2^* = px_1 + (1-p)r$ for any $x_1 \in [0, b]$ (resp. $[0, |l|)$) if $r \leq b < |l|$ (resp. $b \geq |l|$).*

Proof. We show by constructing equilibria where the outlet's strategy β^* and the voter's belief $\mathcal{P}^*$ for off-the-equilibrium-path messages are given by (A.6), (A.7) and (A.8), respectively, and the induced outcome of the news-reporting stage is $\bar{\gamma}_{1/2}$. By the same argument used in the proof of Proposition 2-(i), the optimality of β^* and γ^* except for message $m = \bar{Z}_{12} \cup \bar{Z}_{21}$ can be shown. Hence, it remains to show the optimality of α_i^* and $\gamma^*(\bar{Z}_{12} \cup \bar{Z}_{21})$. Notice that $\bar{\gamma}_{1/2}(z) = (1/2, 1/2)$ holds for any $z \in [0, b]^2$. Hence, for any $\alpha_1 \in \Delta([0, b])^*$, $\bar{U}_1(\alpha_1^*, \alpha_2^*, \bar{\gamma}_{1/2}) = \bar{U}_1(\alpha_1, \alpha_2^*, \bar{\gamma}_{1/2}) = 1-p/2$. That is, candidate 1 has no incentive to deviate. Likewise, because $b \geq r$, $\bar{U}_2(\alpha_1^*, \alpha_2^*, \bar{\gamma}_{1/2}) = \bar{U}_2(\alpha_1^*, \alpha_2, \bar{\gamma}_{1/2}) = 1/2$ for any $\alpha_2 \in \Delta([0, b])^*$. Then, candidate 2 also has no incentive to deviate. Without loss of generality, we assume that $\bar{Z}_{12}(\alpha_1^*, \alpha_2^*) = \{(x_1, px_1 + (1-p)r)\}$ and $\bar{Z}_{21}(\alpha_1^*, \alpha_2^*) = \{(r, px_1 + (1-p)r)\}$. Therefore, message $m = \bar{Z}_{12} \cup \bar{Z}_{21}$ is used on the equilibrium path, and then the consistent belief is as follows: $\mathcal{P}^*((x_1, px_1 + (1-p)r) | \bar{Z}_{12} \cup \bar{Z}_{21}) = p$ and $\mathcal{P}^*((r, px_1 + (1-p)r) | \bar{Z}_{12} \cup \bar{Z}_{21}) = 1-p$. Given $\mathcal{P}^*(\cdot | \bar{Z}_{12} \cup \bar{Z}_{21})$, actions $y = y_1$ and y_2 are indifferent for the voter.

Thus, $\gamma^*(\bar{Z}_{12} \cup \bar{Z}_{21})$ is optimal. Therefore, it is a USE. ■

Lemma 8 *Suppose that $b > r$. If there exists a USE where the induced outcome of the news-reporting stage is $\bar{\gamma}_1$, then $\alpha_2^* = r$ holds.*

Proof. Because the induced outcome of the news-reporting stage is $\bar{\gamma}_1$:

$$\bar{U}_1(\alpha_1^*, \alpha_2^*, \bar{\gamma}_1) = 1 - \frac{1}{2} \sum_{z \in Z_{00}} \Pr(z|\alpha_1^*, \alpha_2^*, \theta_1 = O). \tag{A.39}$$

Hence, to be an equilibrium, $\sum_{z \in Z_{00}} \Pr(z|\alpha_1^*, \alpha_2^*, \theta_1 = O) = 0$ should hold; otherwise, candidate 1 has an incentive to deviate to strategy α_1' such that $S(\alpha_1') \cap S(\alpha_2^*) = \emptyset$. That is, $\sum_{z \in Z_{00}} \Pr(z|\alpha_1^*, \alpha_2^*, \theta_1 = O, \theta_2 = O) = 0$ holds. Because $b > r$:

$$\begin{aligned}\bar{U}_2(\alpha_1^*, \alpha_2^*, \bar{\gamma}_1) &= \frac{1}{2}\left(p \sum_{z \in Z_{00}} \Pr(z|\alpha_1^*, \alpha_2^*, \theta_1 = O, \theta_2 = O) + (1-p) \sum_{z \in Z_{00}} \Pr(z|\alpha_1^*, \alpha_2^*, \theta_1 = I, \theta_2 = O)\right) \\ &= \frac{1}{2}(1-p)\alpha_2^*(r). \end{aligned} \tag{A.40}$$

Because $\bar{U}_2(\alpha_1^*, \alpha_2^*, \bar{\gamma}_{1/2}) \geq \bar{U}_2(\alpha_1^*, \alpha_2, \bar{\gamma}_{1/2})$ holds for any $\alpha_2 \in \Delta(X)^*$, $\alpha_2^*(r) = 1$ should hold. ■

A.5.3 Proof of Theorem 2

(i) First, we suppose that $0 < b \leq r/2$ (Case (i)). Notice that if exactly one of the candidates is the ideological type, then the winner is the opportunistic-type candidate. By Lemma 2-(i), the induced outcome of the news-reporting stage should be $\bar{\gamma}_{1/2}$. If $\bar{Z}_{12}(\alpha_1^*, \alpha_2^*) \cup \bar{Z}_{21}(\alpha_1^*, \alpha_2^*) = \emptyset$, then it should be that $\alpha_1^* = \alpha_2^* = x \in [0, b]$. If $\bar{Z}_{12}(\alpha_1^*, \alpha_2^*) \cup \bar{Z}_{21}(\alpha_1^*, \alpha_2^*) \neq \emptyset$, then $\sum_{x_1 \in [0,b]} \alpha_1^*(x_1)x_1 = \sum_{x_2 \in [0,b]} \alpha_2^*(x_2)x_2$ should hold because of the consistency of belief $\mathcal{P}^*$. Therefore, for USE e:

$$\begin{aligned} d(e) &= p^2 \sum_{x_1 \in [0,b]} \alpha_1^*(x_1)x_1 + p(1-p) \sum_{x_2 \in [0,b]} \alpha_2^*(x_2)x_2 + p(1-p) \sum_{x_1 \in [0,b]} \alpha_1^*(x_1)x_1 \\ &= p \sum_{x_1 \in [0,b]} \alpha_1^*(x_1)x_1 + p(1-p) \sum_{x_2 \in [0,b]} \alpha_2^*(x_2)x_2, \end{aligned} \tag{A.41}$$

which implies that $0 \leq d(e) \leq p(2-p)b$ holds for any equilibrium e. By Lemma 5, there exist $(0,0)$ and (b,b) equilibria whose degrees of distortion are 0 and $p(2-p)b$, respectively.

Second, suppose that $r/2 < b < r$ (Case (ii)). By Lemmas 1 and 3, if exactly one of the candidates is the ideological type, then the winner is the opportunistic-type candidate. By Lemma

2-(ii), if the induced outcome of the news-reporting stage is $\bar{\gamma}_1$, then for any USE e:

$$\begin{aligned} d(e) &= p^2 \sum_{x_1 \in [0,b]} \alpha_1^*(x_1)x_1 + p(1-p) \sum_{x_2 \in [-r+2b,b]} \alpha_2^*(x_2)x_2 + p(1-p) \sum_{x_1 \in [0,b]} \alpha_1^*(x_1)x_1 \\ &= p \sum_{x_1 \in [0,b]} \alpha_1^*(x_1)x_1 + p(1-p) \sum_{x_2 \in [-r+2b,b]} \alpha_2^*(x_2)x_2. \end{aligned} \tag{A.42}$$

As in the same argument used in Case (i), the degree of distortion under $\bar{\gamma}_{1/2}$ is also given by (A.42). Hence, this representation implies that $p(1-p)(-r+2b) \leq d(e) \leq p(2-p)b$ holds for any equilibrium e. By Lemma 6-(i), there exists (b,b) equilibrium whose degree of distortion is $p(2-p)b$. If $p \leq 2/3$, then, by Lemma 6-(ii), there exists $(0,-r+2b)$ equilibrium whose degree of distortion is $p(1-p)(-r+2b)$. Notice that if $p > 2/3$, then $(0,-r+2b)$ equilibrium never exists. However, the degree of distortion of USE e^* constructed in Lemma 6-(ii) where $\alpha_2^* = -r+2b$ is:

$$d(e^*) = p(1-p)(-r+2b) + p\sum_{k=1}^{N} \frac{\varepsilon_k}{N}. \tag{A.43}$$

Because each ε_k can be arbitrarily small, $d(e^*)$ converges to $p(1-p)(-r+2b)$ as $\varepsilon_k \to 0$ for each k. Thus, we can insist that $\underline{D}(b,p) = p(1-p)(-r+2b)$.

Third, we suppose that $r \leq b < |l|$ (Case (iii)). In this scenario, policy pair could be in the disagreement regions $Z_{12} \cup Z_{21}$ even though candidate 1 is the ideological type. By Lemma 2-(ii), the induced outcome of the news-reporting stage is either $\bar{\gamma}_1$ or $\bar{\gamma}_{1/2}$. If the induced outcome of the news-reporting stage is $\bar{\gamma}_1$, then candidate 1 wins with certainty unless policy pairs lie in region Z_{00}. Hence:

$$\begin{aligned} d(e) &= p^2 \sum_{x_1 \in [0,b]} \alpha_1^*(x_1)x_1 + p(1-p)r + p(1-p) \sum_{x_1 \in [0,b]} \alpha_1^*(x_1)x_1 \\ &= p \sum_{x_1 \in [0,b]} \alpha_1^*(x_1)x_1 + p(1-p)r. \end{aligned} \tag{A.44}$$

Next, suppose that the induced outcome of the news-reporting stage is $\bar{\gamma}_{1/2}$. If $\bar{Z}_{12}(\alpha_1^*, \alpha_2^*) \cup \bar{Z}_{21}(\alpha_1^*, \alpha_2^*) = \emptyset$, then $\alpha_1^* = \alpha_2^* = r$ should hold. If $\bar{Z}_{12}(\alpha_1^*, \alpha_2^*) \cup \bar{Z}_{21}(\alpha_1^*, \alpha_2^*) \neq \emptyset$, then the consistent belief implies that $p\sum_{x_1\in[0,b]}\alpha_1^*(x_1)x_1+(1-p)r=\sum_{x_2\in[0,b]}\alpha_2^*(x_2)x_2$. Therefore:

$$\begin{aligned}d(e) &= p^2\left(\frac{1}{2}\sum_{x_1\in[0,b]}\alpha_1^*(x_1)x_1+\frac{1}{2}\sum_{x_2\in[0,b]}\alpha_2^*(x_2)x_2\right)+p(1-p)\left(\frac{1}{2}r+\frac{1}{2}\sum_{x_2\in[0,b]}\alpha_2^*(x_2)x_2\right)\\ &\qquad +p(1-p)\sum_{x_1\in[0,b]}\alpha_1^*(x_1)x_1\\ &=\frac{p}{2}\left((2-p)\sum_{x_1\in[0,b]}\alpha_1^*(x_1)x_1+(1-p)r+\sum_{x_2\in[0,b]}\alpha_2^*(x_2)x_2\right)\\ &=p\sum_{x_1\in[0,b]}\alpha_1^*(x_1)x_1+p(1-p)r. \end{aligned} \tag{A.45}$$

Thus, we can say that $p(1-p)r\le d(e)\le pb+p(1-p)r$. By Lemma 7, there exist $(0,(1-p)r)$ and $(b,pb+(1-p)r)$ equilibria whose degrees of distortion are $p(1-p)r$ and $pb+p(1-p)r$, respectively.

Finally, we suppose that $b\ge|l|$ (Case (iv)). By Lemma 2-(ii), the induced outcome of the news-reporting stage is either $\bar{\gamma}_1$ or $\bar{\gamma}_{1/2}$. Furthermore, by Lemma 4, candidate 1 wins with certainty when $\theta_1=O$ and $\theta_2=I$. Thus, the representation of $d(e)$ is identical to that in Case (iii). Hence, $p(1-p)r\le d(e)$ holds for any equilibrium e. By Lemma 7, there exists $(0,(1-p)r)$ equilibrium whose degree of distortion is $p(1-p)r$. Now, we show that $d(e)<p|l|+(1-p)r$ for any USE e.

Case (iv)-1: $S(\alpha_1^*)\subseteq[0,|l|)$.

Note that:

$$p|l|+p(1-p)r-d(e)=p\left(|l|-\sum_{x_1\in[0,b]}x_1\alpha_1^*(x_1)\right)>0, \tag{A.46}$$

where the last inequality comes from $S(\alpha_1^*)\subseteq[0,|l|)$.

Case (iv)-2: $S(\alpha_1^*)\cap[|l|,b]\ne\emptyset$.

By Lemmas 4 and 8, we can restrict our attention to the scenario where the induced outcome of the news-reporting stage is $\bar{\gamma}_1$ and $\alpha_2^*=r$ without loss of generality. To support $\bar{\gamma}_1$ in equilibrium, the following condition should hold:

$$\begin{aligned}
&p^2 \sum_{x_1 \in [0,b]} x_1 \alpha_1^*(x_1) + p(1-p)r + p(1-p) \sum_{x_1 \in [|l|,b]} x_1 \alpha_1^* \\
&\qquad < p \sum_{x_2 \in [0,b]} x_2 \alpha_2^*(x_2) + p(1-p)|l| \sum_{x_1 \in [|l|,b]} \alpha_1^*(x_1). \\
\Longleftrightarrow\ & p \sum_{x_1 \in [0,b]} x_1 \alpha_1^*(x_1) + (1-p) \sum_{x_1 \in [|l|,b]} x_1 \alpha_1^* < pr + (1-p)|l| \sum_{x_1 \in [|l|,b]} \alpha_1^*(x_1). \qquad \text{(A.47)}
\end{aligned}$$

Now, we show that $\sum_{x_1 \in [0,b]} x_1 \alpha_1^*(x_1) < |l|$. Suppose, in contrast, that $\sum_{x_1 \in [0,b]} x_1 \alpha_1^*(x_1) \geq |l|$. Then:

$$\begin{aligned}
p|l| + (1-p) \sum_{x_1 \in [|l|,b]} x_1 \alpha_1^*(x_1) &\leq p \sum_{x_1 \in [0,b]} x_1 \alpha_1^*(x_1) + (1-p) \sum_{x_1 \in [|l|,b]} x_1 \alpha_1^*(x_1) \\
&< pr + (1-p)|l| \sum_{x_1 \in [|l|,b]} \alpha_1^*(x_1), \qquad \text{(A.48)}
\end{aligned}$$

where the last inequality comes from (A.47). Because $\sum_{x_1 \in [|l|,b]} x_1 \alpha_1^*(x_1) \geq |l| \sum_{x_1 \in [|l|,b]} \alpha_1^*(x_1)$, $|l| < r$ should hold for satisfying (A.48), which is a contradiction. Thus, $\sum_{x_1 \in [0,b]} x_1 \alpha_1^*(x_1) < |l|$ holds. Hence:

$$p|l| + p(1-p)r - d(e) = p \left(|l| - \sum_{x_1 \in [0,b]} x_1 \alpha_1^*(x_1) \right) > 0. \qquad \text{(A.49)}$$

By Lemma 7, for any $x_1 \in [0, |l|)$, there exists $(x_1, px_1 + (1-p)r)$ equilibrium whose degree of distortion is $px_1 + p(1-p)r$. Because this value converges to $p|l| + p(1-p)r$ as $x_1 \to |l|$, we can conclude that $\overline{D}(b,p) = p|l| + p(1-p)r$.

(ii) The necessity is straightforward from (i) of this theorem. It then reminds to show the sufficiency. First, suppose that $0 < b \leq r/2$ (Case (i)), and fix $d \in [0, p(2-p)b]$, arbitrarily. By Lemma 5, there exists USE e^* such that $\alpha_1^* = \alpha_2^* = d/(p(2-p)) \in [0,b]$. By simple algebra:

$$d(e^*) = p^2 \left(\frac{d}{p(2-p)} \right) + p(1-p) \left(\frac{d}{p(2-p)} \right) + p(1-p) \left(\frac{d}{p(2-p)} \right) = d. \qquad \text{(A.50)}$$

Second, we suppose that $r/2 < b < r$ (Case (ii)).

Case (ii)-1: $p \leq 2/3$.

Fix $d \in [p(1-p)(-r+2b), p(2-p)(-r+2b))$, arbitrarily. By Lemma 6-(ii), there exists USE

e^* such that $\alpha_1^* = d/p - (1-p)(-r+2b)$ and $\alpha_2^* = -r+2b$. By simple algebra:

$$d(e^*) = p^2\left(\frac{d}{p} - (1-p)(-r+2b)\right) + p(1-p)(-r+2b) + p(1-p)\left(\frac{d}{p} - (1-p)(-r+2b)\right)$$
$$= d. \tag{A.51}$$

For $d \in [p(2-p)(-r+2b), pb + p(1-p)r]$, we can construct a desired USE by the similar argument used in Case (i), whose existence is guaranteed by Lemma 6-(i).

Case (ii)-2: $p > 2/3$.

Fix $d \in (p(1-p)(-r+2b), p(2-p)(-r+2b))$, arbitrarily. Let N be the integer such that $2(N-1)/(1+2(N-1)) < p \leq 2N/(1+2N)$. If N is an even number, then we consider USE e^* constructed in Lemma 6-(ii) such that

- $S(\alpha_1^*) = \{\, x+\delta, x-\delta, x+2\delta, x-2\delta, \cdots, x+(N/2)\delta, x-(N/2)\delta \,\}$ where $x \equiv d/p-(1-p)(-r+2b)$, and $\delta > 0$ is so small that $x_1 \in (0, -r+2b)$ for any $x_1 \in S(\alpha_1^*)$;
- $\alpha_2^* = -r+2b$.

By simple algebra:

$$d(e^*) = p^2\left(\sum_{x_1 \in S(\alpha_1^*)} \frac{x_1}{N}\right) + p(1-p)(-r+2b) + p(1-p)\left(\sum_{x_1 \in S(\alpha_1^*)} \frac{x_1}{N}\right) = d. \tag{A.52}$$

If N is an odd number, then we can construct the desired USE with the same structure except for that $S(\alpha_1) = \{\, x, x+\delta, x-\delta, \cdots, x+((N-1)/2)\delta, x-((N-1)/2)\delta \,\}$. For $d \in [p(2-p)(-r+2b), pb+p(1-p)r]$, we can construct a desired USE by the same argument used in Case (ii)-1, whose existence is guaranteed by Lemma 6-(i).

Finally, we assume that $b \geq r$ (Cases (iii) and (iv)), and fix $d \in [p(1-p)r, pb+p(1-p)r]$ (resp. $[p(1-p)r, p|l| + p(1-p)r)$) arbitrarily if $r \leq b < |l|$ (resp. $b \geq |l|$). We consider USE e^* such that $\alpha_1^* = d/p - (1-p)r$ and $\alpha_2^* = d + (1-p)^2 r$ whose existence is guaranteed by Lemma 7. Then:

$$d(e^*) = p^2\left(\frac{d}{p} - (1-p)r\right) + p(1-p)r + p(1-p)\left(\frac{d}{p} - (1-p)r\right) = d. \tag{A.53}$$

■

A.6 Proofs of Corollaries 1 and 2

They are straightforward from Theorem 2 and equation (13). ■

A.7 Proof of Claim 2

First, suppose that $\tilde{x}_1 > x_2$, and then we show that the following is a USE. The outlet's strategy β^* is given by:

$$\beta^*(m) = \begin{cases} \bar{Z}_{12} \cup \bar{Z}_{21} & \text{if } z \in \bar{Z}_{12} \cup Z_{21}, \\ z & \text{otherwise,} \end{cases} \tag{A.54}$$

and the voter's belief for off-the-equilibrium-path messages is given by:

$$S(\mathcal{P}^*(z|m)) \subseteq \begin{cases} m \cap Z_{21} & \text{if } m \cap Z_{21} \neq \emptyset, \\ m \cap Z_{01} & \text{if } m \cap Z_{21} = \emptyset \text{ and } m \cap Z_{01} \neq \emptyset, \\ m & \text{otherwise.} \end{cases} \tag{A.55}$$

Notice that because $\tilde{x}_1 > x_2$, $\gamma^*(\bar{Z}_{12} \cup \bar{Z}_{21}) = (0, 1)$. We can show the optimality of β^* by the similar argument used in the proof of Lemma 6. Thus, it is a USE. Therefore, the degree of distortion $d(\alpha_1, x_2)$ is:

$$d(\alpha_1, x_2) = p^2 x_2 + p(1-p)x_2 + p(1-p)\hat{x}_1 = px_2 + p(1-p)\hat{x}_1. \tag{A.56}$$

Second, suppose that $\tilde{x}_1 < x_2$, and then show that the following is a USE: the outlet's strategy β^* and the voter's belief $\mathcal{P}^*$ for off-the-equilibrium-path messages are given by (A.33) and (A.34), respectively. Notice that because $\tilde{x}_1 < x_2$, $\gamma^*(\bar{Z}_{12} \cup \bar{Z}_{21}) = (1, 0)$. By the same argument used in the proof of Lemma 6, we can show the optimality of β^*. Thus, it is a USE. Therefore, the degree of distortion $d(\alpha_1, x_2)$ is:

$$d(\alpha_1, x_2) = p^2 \hat{x}_1 + p(1-p)r + p(1-p)\hat{x}_1 = p\hat{x}_1 + p(1-p)r. \tag{A.57}$$

Finally, suppose that $\tilde{x}_1 = x_2$. By the same argument used in the proof of Proposition 2, we can show that there exists a USE in which β^* and $\mathcal{P}^*$ are given by (A.6), (A.7) and (A.8), respectively. Because $\tilde{x}_1 = x_2$, the degree of distortion is identical to that for $\tilde{x}_1 > x_2$. ■

A.8 Proof of Proposition 3

(i) Suppose that $\hat{x}_1 \neq 0$. As in the proof of Claim 2, the outlet's equilibrium strategy and the voter's off-the-equilibrium-path belief is given by (A.54) and (A.55), respectively. Furthermore, it is straightforward that the voter's following response to the suppressed message $m = \bar{Z}_{12} \cup \bar{Z}_{21}$ is optimal:

$$\gamma^*(\bar{Z}_{12} \cup \bar{Z}_{21}) = \begin{cases} (1,0) & \text{if } \tilde{x}_1 < \hat{x}_2, \\ (1/2,1/2) & \text{if } \tilde{x}_1 = \hat{x}_2, \\ (0,1) & \text{if } \tilde{x}_1 > \hat{x}_2, \end{cases} \tag{A.58}$$

where $\hat{x}_2 \equiv \sum_{x_2 \in X} x_2 \alpha_2^*(x)_2$. Hence, it is sufficient to show that for any $x_2' \in [0, \tilde{x}_1]$, there exists an incentive compatible strategy α_2^* such that $\hat{x}_2 = x_2'$ given α_1, β^*, and γ^*. Fix $x_2' \in [0, \tilde{x}_1)$, arbitrarily. If $x_2' \notin (S(\alpha_1) \cup \{r\})$, then $\alpha_2^* = x_2'$. That is, candidate 2's winning probability from this strategy is 1, and then he has no incentive to deviate. Suppose that $x_2' \in (S(\alpha_1) \cup \{r\})$. By construction, there exists $\delta > 0$ such that $\{x_2' - \delta, x_2' + \delta\} \cap (S(\alpha_1) \cup \{r\}) = \emptyset$. Then, define α_2^* as a mixed strategy such that $\alpha_2^*(x_2' - \delta) = \alpha_2^*(x_2' + \delta) = 1/2$. Notice that his winning probability from this mixed strategy is 1, and then he has no incentive to deviate. Therefore, we can construct a desired USE. For $x_2' = \tilde{x}_1$, it is obvious that candidate 2 has no incentive to deviate because his winning probability is $1/2$ under any policy pair in $[0, b]^2$. Next, suppose that $\hat{x}_1 = 0$; that is, $\alpha_1 = 0$. We can construct a desired USE for any $x_2' \in (0, p(1-p)r]$ by the same argument used above. Furthermore, by the same argument used in the proof of Theorem 1, $\alpha_2^* = 0$ cannot be supported in equilibrium.

(ii) It is straightforward that from the facts that $\sup D_{nc}(\alpha_1, p)$ is increasing in $\hat{x}_1$, and $\hat{x}_1 \leq b$.

(iii) It is obvious from (i). ■

A.9 Proof of Proposition 4

(i) (Existence) We show that there exists a $(0,0)$ equilibrium in which the front-runner is candidate 1. First, we show the optimality of $\gamma^*(\phi) = (1,0)$. Given α_1^*, α_2^*, and $m = \phi$, the voter's expected utility from actions $y = y_1$ and y_2 are $-(1-p)r$ and $-(1-p)|l|$, respectively. Hence, because $r < |l|$, we say that $\gamma^*(\phi) = (1,0)$ is the voter's best response. Next, we show the optimality of α_1^*. Given α_2^* and $\bar{\gamma}_1$, candidate 1's equilibrium utility is $\bar{U}_1(\alpha_1^*, \alpha_2^*, \bar{\gamma}_1) = p(1+q)/2 + (1-p)$. If candidate 1 deviates to $\alpha_1 = x_1' \neq 0$, then his payoff is $\bar{U}_1(x_1', \alpha_2^*, \bar{\gamma}_1) = pq + (1-p)$. Because $q < (1+q)/2$, candidate 1 has no incentive to deviate. Finally, we show the optimality of α_2^*. Given α_1^* and $\bar{\gamma}_1$, candidate 2's equilibrium payoff is $\bar{U}_2(\alpha_1^*, \alpha_2^*, \bar{\gamma}_1) = (1-q)(1-p/2)$. If candidate 2 deviates to $\alpha_2 = x_2' \neq 0$, then his payoff is $\bar{U}_2(\alpha_1^*, x_2', \bar{\gamma}_1) = (1-q)(1-p)$. Because $1 - p < 1 - p/2$, candidate 2 has no incentive to deviate. Therefore, we say that there exists a $(0,0)$ equilibrium.

(Uniqueness) Suppose, in contrast, that there exists an equilibrium in which either $\alpha_1^* \neq 0$ or $\alpha_2^* \neq 0$. Without loss of generality, we assume that $\alpha_1^* \neq 0$. That is, there exists $x_1' \in S(\alpha_1^*)$ such that $x_1' \neq 0$. Furthermore, we assume that $\gamma^*(\phi) = (\eta, 1-\eta)$ where $\eta \in \{0, 1/2, 1\}$. First, suppose that $0 \notin S(\alpha_2^*)$. In this scenario, if candidate 1 proposes policy $x_1 = 0$ for certain, then his payoff is $\bar{U}_1(0, \alpha_2^*, \bar{\gamma}_\eta) = (1-q) + q\eta$. Hence, $\bar{U}_1(x_1, \alpha_2^*, \bar{\gamma}_\eta) \leq (1-q) + q\eta$ holds for any $x_1 \in [0, b]$. Therefore, to hold this equilibrium, $x_1 < x_2$ holds for any $x_1 \in S(\alpha_1^*)$ and $x_2 \in S(\alpha_2^*)$; otherwise, candidate 1 deviates to $\alpha_1 = 0$. In other words, candidate 2 never wins when candidate 1 is the opportunistic type. Thus, candidate 2's equilibrium payoff is $\bar{U}_2(\alpha_1^*, \alpha_2^*, \bar{\gamma}_\eta) \leq (1-q)(1-p) + q(1-\eta)$. However, if candidate 2 deviates to $\alpha_2 = 0$, then:

$$\begin{aligned}\bar{U}_2(\alpha_1^*, 0, \bar{\gamma}_\eta) &= (1-q)\left(p\left(\frac{1}{2}\alpha_1^*(0) + (1-\alpha_1^*(0))\right) + (1-p)\right) + q(1-\eta) \\ &> (1-q)(1-p) + q(1-\eta) \geq \bar{U}_2(\alpha_1^*, \alpha_2^*, \bar{\gamma}_\eta). \end{aligned} \tag{A.59}$$

That is, candidate 2 has an incentive to deviate, which is a contradiction. Therefore, $0 \in S(\alpha_2^*)$ should hold. Notice that because $x' \in S(\alpha_1^*)$, candidate 1's equilibrium payoff is:

$$\bar{U}_1(\alpha_1^*, \alpha_2^*, \bar{\gamma}_\eta) \leq (1-q)\left(p\left(\frac{1}{2}\alpha_2^*(x_1') + \sum_{x_2 > x_1'} \alpha_2^*(x_2)\right) + (1-p)\right) + q\eta, \tag{A.60}$$

where the strict inequality holds when $x' \geq |l|$. Now, if candidate 1 deviates to $\alpha_1 = 0$, then $\bar{U}_1(0, \alpha_2^*, \bar{\gamma}_\eta) = (1-q)(p(\alpha_2^*(0)/2 + (1-\alpha_2^*(0))) + (1-p)) + q\eta$. However:

$$\bar{U}_1(0, \alpha_2^*, \bar{\gamma}_\eta) - \bar{U}_1(\alpha_1^*, \alpha_2^*, \bar{\gamma}_\eta) \geq (1-q)p\left(\frac{1}{2}(\alpha_2^*(0) + \alpha_2^*(x_1')) + \sum_{0 < x_2 < x_1'} \alpha_2^*(x_2)\right) > 0, \tag{A.61}$$

where the first and second inequalities come from (A.60) and $\alpha_2^*(0) > 0$, respectively. That is, candidate 1 has an incentive to deviate, which is a contradiction. Thus, $(0, 0)$ equilibrium is the unique equilibrium.

(ii) Because $(0, 0)$ equilibrium is the unique equilibrium, it is straightforward that the degree of distortion is $d(q) = p(1-p)qr$. ■

Appendix B: Supplementary Materials (Not for Publication)

B.1 Omitted Proofs

B.1.1 Proof of Proposition 1

(i) (Existence) Because there is no media manipulation, $\beta^*(z) = z$ for any $z \in Z$. Hence, it is sufficient to show that the following is a PBE: $\alpha_1^* = \alpha_2^* = 0$ and $\gamma^*(z) = y^v(z)$ because there is no media manipulation. It is straightforward that γ^* is undominated and optimal for the voter. Given γ^* and α_2^*, strategy α_1^* is optimal for candidate 1; that is, the winning probability from α_1^* is $1/2$, but his winning probabilities from other strategies are less than $1/2$. The same argument holds for candidate 2. Thus, it is a PBE.

(Uniqueness) It is obvious that for any policy pair z, the voter's unique undominated strategy is given by $\gamma^*(z) = y^v(z)$. Then, suppose, in contrast, that there exists an equilibrium where either $\alpha_1^* \neq 0$ or $\alpha_2^* \neq 0$ holds. Without loss of generality, assume that $\alpha_1^* \neq 0$. However, candidate 1 can strictly improve his winning probability by proposing policy 0 for certain whatever candidate 2's strategy is, which is a contradiction. Therefore, the $(0, 0)$ equilibrium is the unique one.

(ii) It is obvious from Table 1 in the body of the paper. ■

B.1.2 Proof of Proposition 2-(ii)

Suppose, in contrast, that there exists PBE $(\alpha_1^*, \alpha_2^*, \beta^*, \gamma^*; \mathcal{P}^*)$ such that $\gamma^*(\beta^*(z)) = y^v(z)$ for any $z \in Z$. Fix $z \in Z_{12}$ and $z' \in Z_{21}$, arbitrarily. Hence, $\gamma^*(\beta^*(z)) = (1, 0)$ and $\gamma^*(\beta^*(z')) = (0, 1)$. To be incentive compatible, the following conditions must hold: (1) $\gamma^*(m) = (0, 1)$ for any message $m \in M(z)$, and (2) $\gamma^*(m') = (1, 0)$ for any message $m' \in M(z')$. However, there is no incentive compatible reaction to message $m = \{z, z'\} \in M(z) \cap M(z')$, which is a contradiction. ■

B.2 Justification for the USE

B.2.1 Certifiable dominance

In this subsection, we formally define the notion of certifiable dominance, and provide some related results. The following analysis is based on Miura (2016). Let $B^0 \equiv \{\beta \in \Delta(M)^Z \mid S(\beta(z)) \subseteq$

$M(z)$ for any $z \in Z\}$. Define:

$$W(z, \gamma(m)) \equiv \sum_{y \in Y} w(z, y) \Pr(y|\gamma(m)), \tag{B.1}$$

$$\Omega(\alpha_1, \alpha_2, \beta, \gamma) \equiv \sum_{z \in Z(\alpha_1, \alpha_2)} \sum_{m \in M} W(z, \gamma(m)) \Pr(m|\beta(z)) \Pr(z|\alpha_1, \alpha_2). \tag{B.2}$$

Furthermore, for $i, i' \in \{1, 2\}$ with $i \neq i'$, let $Z_{i0} \equiv \{z \in Z_0 \mid v(z, y_i) > v(z, y_{i'})\}$ be the set of policy pairs in which the voter strictly prefers candidate i, but the candidates are indifferent for the outlet.

Definition B.1 ***Certifiable dominance (Miura, 2016)***
The outlet's strategy $\beta \in B^0$ is certifiably undominated if it is (weakly) undominated in $\Delta(X)^{*2} \times B^0 \times \Gamma$.

The certifiable dominance is a modified version of the weak dominance that is consistent with the rationality of the players and the assumption that the outlet's private information is fully certifiable. Because the voter knows that the outlet must contain the true policy pair in messages, he seems not to choose an action that is suboptimal under any state in the observed message. That is, a reasonable strategy of the rational voter should be in Γ. Furthermore, because the outlet knows that the voter is rational and he understands the message structure, it seems reasonable to restrict the voter's strategies to Γ when we apply the weak dominance argument to the outlet's strategies.[57] First, we can say that the outlet who observes policy pairs in the agreement regions never send "careless" messages that could induce her unfavorite outcome with positive probability.

Lemma B.1 *(Corollary of Proposition 3 of Miura (2016))*
If the outlet's strategy β is certifiably undominated, then $m \subseteq Z_{11} \cup Z_{12} \cup Z_{10}$ (resp. $Z_{22} \cup Z_{21} \cup Z_{20}$) for any $z \in Z_{11}$ (resp. Z_{22}) and $m \in S(\beta(z))$.

Proof. Suppose, in contrast, that strategy β is certifiably undominated, but there exist $z' \in Z_{11}$ and $m' \in S(\beta(z'))$ such that $m' \nsubseteq Z_{11} \cup Z_{12} \cup Z_{10}$. Now, we consider the following strategy β^* defined by:

$$\beta^*(z) \equiv \begin{cases} z' & \text{if } z = z', \\ \beta(z) & \text{otherwise.} \end{cases} \tag{B.3}$$

[57] The notion of certifiable dominance is closely related to Δ-rationalizability and prudent rationalizability developed by Battigalli and Siniscalchi (2003) and Heifetz et al. (2011), respectively. See Miura (2016, 2018) for the details.

Note that if $\gamma \in \Gamma$, then $S(\gamma(m')) \subseteq \{y_1, y_2\}$. Fix α_1, α_2 and $\gamma \in \Gamma$, arbitrarily. Then:

$$\Omega(\alpha_1, \alpha_2, \beta^*, \gamma) - \Omega(\alpha_1, \alpha_2, \beta, \gamma) = \left\{ w(z', y^v(z')) - \sum_{m \in M} W(z', \gamma(m))\Pr(m|\beta(z)) \right\} \Pr(z'|\alpha_1, \alpha_2). \tag{B.4}$$

If either $\Pr(z'|\alpha_1, \alpha_2) = 0$ or $\Pr(z'|\alpha_1, \alpha_2) \neq 0$ with $S(\gamma(m')) = \{y_1\}$ holds, then (B.4) is 0. Then, we suppose that $\Pr(z'|\alpha_1, \alpha_2) \neq 0$, and either $S(\gamma(m')) = \{y_2\}$ or $\{y_1, y_2\}$. Because $z' \in Z_{11}$, $w(z', y^v(z')) - \sum_{m \in M} W(z', \gamma(m))\Pr(m|\beta(z')) > 0$ holds. Hence, (B.4) is positive. That is, strategy β is certifiably dominated by strategy β^*, which is a contradiction. Likewise, we can show that $m \subseteq Z_{22} \cup Z_{21} \cup Z_{20}$ holds for any $z \in Z_{22}$ and $m \in S(\beta(z))$. ■

Now, we impose the following additional requirement upon off-the-equilibrium-path beliefs.

Requirement B.1 *If $\beta^{-1}(m) \neq \emptyset$, then $S(\mathcal{P}(\cdot|m)) \subseteq \beta^{-1}(m)$ holds.*

This requirement means that if there exist policy pairs in which message m is sent under strategy β, then the posterior belief after observing message m must be a probability distribution over the set of such policy pairs even though that message is sent off the equilibrium path. Intuitively, we require that the voter's belief is consistent with Bayes' rule as much as possible. Notice that the consistency is automatically satisfied if message m is sent on the equilibrium path, i.e., for message m such that $m \in S(\beta^*(z))$ for some policy pair $z \in Z(\alpha_1^*, \alpha_2^*)$. However, it might not hold for message m' sent by the outlet who observes policy pairs off the equilibrium path, i.e., for message m' such that $m' \in S(\beta^*(z'))$ for some policy pair $z' \notin Z(\alpha_1^*, \alpha_2^*)$. This requirement extends that consistency up to the later scenario. We call a PBE satisfying Requirement B.1 *strong perfect Bayesian equilibrium* (hereafter, SPBE), which is defined as follows.

Definition B.2 ***SPBE***
A PBE $(\alpha_1^*, \alpha_2^*, \beta^*, \gamma^*; \mathcal{P}^*)$ is an SPBE if $\mathcal{P}^*$ satisfies Requirement B.1.[58]

Once we focus on SPBEs where the outlet's strategy is certifiably undominated, the equilibrium outcomes over the disagreement regions must be constant as shown in the following proposition. It is worthwhile emphasizing that those equilibrium outcomes can be replicated by PBEs where the

[58] The SPBE is associated with the PBE defined in Fudenberg and Tirole (1991) and Gibbons (1992). Because the candidates' actions are unobservable to the voter and there is no prior distribution over policy pairs, the posterior belief after observing message m' mentioned above is not uniquely pinned down.

outlet's strategy is simple. Hence, without loss of generality in the above sense, we can restrict our attention to PBEs in which the outlet's strategy is simple.

Proposition B.1 *(Corollary of Proposition 4 of Miura (2016))*
If $(\alpha_1^*, \alpha_2^*, \beta^*, \gamma^*; \mathcal{P}^*)$ is an SPBE where β^* is certifiably undominated, then $\gamma^*(\beta^*(z)) = \gamma^*(\beta^*(z'))$ holds for any $z, z' \in Z_{12} \cup Z_{21}$.

Proof. Suppose, in contrast, that there exists SPBE $(\alpha_1^*, \alpha_2^*, \beta^*, \gamma^*; \mathcal{P}^*)$ where there exist $z, z' \in Z_{12} \cup Z_{21}$ such that $\gamma^*(\beta^*(z)) \neq \gamma^*(\beta^*(z'))$. Let $\beta^*(z) \equiv m$ and $\beta^*(z') \equiv m'$.[59] First, suppose the scenario where $z, z' \in Z_{12}$ or Z_{21}. Without loss of generality, we assume that $z, z' \in Z_{12}$.

Case (i): $\gamma^*(\beta^*(z)) = (1, 0)$.

To hold this equilibrium, $\gamma^*(m) = (1, 0)$ should hold for any $m \in M(z)$. Hence, $\gamma^*(\beta^*(z'')) = (1, 0)$ must hold for any $z'' \in Z_{21}$. Without loss of generality, assume that $\gamma^*(\beta^*(z')) = (0, 1)$. Hence, by Requirement B.1, $\beta^{-1}(m') \cap (Z_{22} \cup Z_{21}) \neq \emptyset$ holds. Because of $\beta^{-1}(m') \cap Z_{22} = \emptyset$ by Lemma B.1, $\beta^{-1}(m') \cap Z_{21} \neq \emptyset$ must hold. However, because $\gamma^*(\beta^*(z'')) = (1, 0)$ holds for any $z'' \in Z_{21}$, $\beta^{-1}(m') \cap Z_{21} = \emptyset$, which is a contradiction.

Case (ii): $\gamma^*(\beta^*(z)) = (0, 1)$.

By Requirement B.1 and Lemma B.1, $\beta^{-1}(m) \cap Z_{21} \neq \emptyset$ must hold. That is, there exists $\hat{z} \in Z_{21}$ such that $\beta^*(\hat{z}) = m$, which induces that $\gamma^*(\beta^*(\hat{z})) = (0, 1)$. To hold this equilibrium, $\gamma^*(\hat{m}) = (0, 1)$ should hold for any $\hat{m} \in M(\hat{z})$. However, because $\gamma^*(\beta^*(z')) \neq (0, 1)$, the outlet observing policy pair z' has an incentive to send a message including policy pair $\hat{z}$, which is contradiction.

Case (iii): $\gamma^*(\beta^*(z)) = (1/2, 1/2)$.

We can derive a contradiction as in Cases (i) and (ii).

Next, suppose that $z \in Z_{12}$ and $z' \in Z_{21}$. By Proposition 2-(ii), either $\gamma^*(\beta^*(z)) \neq (1, 0)$ or $\gamma^*(\beta^*(z')) \neq (0, 1)$ holds. Without loss of generality, assume that $\gamma^*(\beta^*(z)) = (0, 1)$.[60] By the similar argument as in Case (ii), there exists $\hat{z} \in Z_{21}$ such that $\gamma^*(\hat{m}) = (0, 1)$ holds for any $\hat{m} \in M(\hat{z})$. Suppose that $\gamma^*(\beta^*(z')) = (1, 0)$. By Requirement B.1 and Lemma B.1, $\beta^{-1}(m') \cap Z_{12} \neq \emptyset$ holds. That is, there exists $\tilde{z} \in Z_{12}$ such that $\beta^*(\tilde{z}) = m'$, which induces that $\gamma^*(\beta^*(\tilde{z})) = (1, 0)$. However, the outlet observing policy pair $\tilde{z}$ has an incentive to deviate to a message containing policy pair $\hat{z}$, which is a contradiction. Likewise, we can derive a contradiction in the scenario where $\gamma^*(\beta^*(z')) = (1/2, 1/2)$. ■

[59] Without loss of generality, we can restrict our attention to the scenario where the outlet adopts a pure strategy.
[60] We can derive contradictions in other cases by the similar argument used here.

B.2.2 Undominated strategies of the candidates

In this subsection, we characterize the set of undominated strategies of the candidates under the restriction mentioned in the body of the paper. We say that strategy $\alpha_i \in \Delta(X)^*$ is *weakly dominated with respect to* $\bar{\Gamma}$ by strategy $\alpha_i' \in \Delta(X)^*$ if $\bar{U}_i(\alpha_i', \alpha_j, \bar{\gamma}_c) \geq \bar{U}_i(\alpha_i, \alpha_j, \bar{\gamma}_c)$ holds for any $\alpha_j \in \Delta(X)^*$ and $\bar{\gamma}_c \in \bar{\Gamma}$ with strict inequality for some α_j' and $\bar{\gamma}_c'$.[61] We say that strategy $\alpha_i \in \Delta(X)^*$ is *undominated with respect to* $\bar{\Gamma}$ if it is not weakly dominated by other strategies. Let A_i be the set of candidate i's undominated strategies with respect to $\bar{\Gamma}$, and $A \equiv A_1 \times A_2$. The set of undominated strategies A_i can be characterized as follows. Notice that this characterization is irrelevant to the magnitude of the preference bias.

Proposition B.2 *$A_i = \Delta([0, b])^*$ holds for any $i \in \{1, 2\}$.*

Proof. ($A_i \subseteq \Delta([0, b])^*$) Suppose, in contrast, that there exists $\alpha_i \in A_i$ such that $\alpha_i \notin \Delta([0, b])^*$. That is, there exists policy $x_i' \in S(\alpha_i)$ such that $x_i' \in [-\bar{x}, 0) \cup (b, \bar{x}]$. Without loss of generality, we assume that $i = 1$. First, we suppose that $x_1' \in [-\bar{x}, 0)$. Without loss of generality, we assume that $|x_1'| < 2b$. Consider the following strategy $\hat{\alpha}_1$ defined by:

$$\hat{\alpha}_1(x_1) \equiv \begin{cases} 0 & \text{if } x_1 = x_1', \\ \alpha_1(0) + \alpha_1(x_1') & \text{if } x_1 = 0, \\ \alpha_1(x_1) & \text{otherwise.} \end{cases} \tag{B.5}$$

It is sufficient to show that $\mu_1(0, x_2) \geq \mu_1(x_1', x_2)$ for any $x_2 \in X$ and any $\bar{\gamma}_c \in \bar{\Gamma}$ with strictly inequality for some x_2' and $\bar{\gamma}_c'$.

(1) If $x_2 \in (2b, \bar{x})$, then $(0, x_2) \in \cup Z_{11}$. Hence, $\mu_1(0, x_2) = 1 \geq \mu_1(x_1', x_2)$ for any $\bar{\gamma}_c \in \bar{\Gamma}$.

(2) If $x_2 = 2b$, then $(0, 2b) \in Z_{10}$ and $(x_1', 2b) \in Z_{12}$. Hence, $\mu_1(0, 2b) \geq \mu_1(x_1', 2b)$ for any $\bar{\gamma}_c \in \bar{\Gamma}$.

(3) If $x_2 \in (-x_1', 2b)$, then $(0, x_2), (x_1', x_2) \in Z_{12}$. Hence, $\mu_1(0, x_2) = \mu_1(x_1', x_2)$ for any $\bar{\gamma}_c \in \bar{\Gamma}$.

(4) If $x_2 = -x_1'$, then $(0, x_2) \in Z_{12}$ and $(x_1', x_2) \in Z_{02}$. Note that if $c \leq 1/2$, then $\bar{\gamma}_c(x_1', x_2) = (c, 1 - c)$. Hence, $\mu_1(0, x_2) = \mu_1(x_1', x_2) = c$. If $c > 1/2$, then $\bar{\gamma}_c(x_1', x_2) = (1/2, 1/2)$. Hence, $\mu_1(0, x_2) = c > \mu_1(x_1', x_2) = 1/2$.

(5) If $x_2 \in [0, -x_1')$, then $(x_1', x_2) \in Z_{22}$. Hence, $\mu_1(0, x_2) \geq \mu_1(x_1', x_2) = 0$ for any $\bar{\gamma}_c \in \bar{\Gamma}$.

(6) If $x_2 \in (x_1', 0)$, then $(0, x_2) \in Z_{11}$ and $(x_1', x_2) \in Z_{22}$. Hence, $\mu(0, x_2) > \mu_1(x_1', x_2)$ for any $\bar{\gamma}_c \in \bar{\Gamma}$.

[61] Definition of $\bar{\gamma}_c$, $\bar{\Gamma}$ and U_i is in Appendix A.

(7) If $x_2 \in [-\bar{x}, x_1']$, then $(0, x_2) \in Z_{11}$. Hence, $\mu_1(0, x_2) \geq \mu_1(x_1', x_2)$ for any $\bar{\gamma}_c \in \bar{\Gamma}$.

That is, strategy α_1 is weakly dominated with respect to $\bar{\Gamma}$ by strategy $\hat{\alpha}_1$, which is a contradiction.

Next, suppose that $x_1' \in (b, \bar{x}]$. Without loss of generality, we assume that $|x_1'| < 3b$. Similar to the above case, we consider the following strategy $\hat{\alpha}_1$ defined by:

$$\hat{\alpha}_1(x_1) \equiv \begin{cases} 0 & \text{if } x_1 = x_1', \\ \alpha_1(b) + \alpha_1(x_1') & \text{if } x_1 = b, \\ \alpha_1(x_1) & \text{otherwise.} \end{cases} \tag{B.6}$$

It is sufficient to show that $\mu_1(b, x_2) \geq \mu_1(x_1', x_2)$ for any $x_2 \in X$ and any $\bar{\gamma}_c$ with strict inequality for some $x_2' \in X$ and $\bar{\gamma}_c'$.

(1) If $x_2 \in [x_1', \bar{x}]$, then $(b, x_2) \in Z_{11}$. Hence, $\mu_1(b, x_2) = 1 \geq \mu_1(x_1', x_2)$ for any $\bar{\gamma}_c \in \bar{\Gamma}$.

(2) If $x_2 \in (b, x_1')$, then $(b, x_2) \in Z_{11}$ and $(x_1', x_2) \in Z_{22}$. Hence, $\mu_1(b, x_2) = 1 > \mu_1(x_1', x_2) = 0$ for any $\bar{\gamma}_c \in \bar{\Gamma}$.

(3) If $x_2 \in (-x_1' + 2b, b]$, then $(x_1', x_2) \in Z_{22}$. Hence, $\mu_1(b, x_2) \geq \mu_1(x_1', x_2) = 0$ for any $\bar{\gamma}_c \in \bar{\Gamma}$.

(4) If $x_2 = -x_1' + 2b$, then $(b, -x_1' + 2b) \in Z_{21}$ and $(x_1', -x_1' + 2b) \in Z_{20}$. Hence, $\mu_1(b, x_2) \geq \mu_1(x_1', x_2) = 0$ for any $\bar{\gamma}_c \in \bar{\Gamma}$.

(5) If $x_2 \in (-b, -x_1' + 2b)$, then $(b, x_2), (x_1', x_2) \in Z_{21}$. Hence, $\mu_1(b, x_2) = \mu_1(x_1', x_2)$ for any $\bar{\gamma}_c \in \bar{\Gamma}$.

(6) If $x_2 = -b$, then $(b, x_2) \in Z_{01}$ and $(x_1', x_2) \in Z_{21}$. Note that if $c \geq 1/2$, then $\bar{\gamma}_c(b, x_2) = (c, 1 - c)$. Hence, $\mu_1(b, x_2) = \mu_1(x_1', x_2) = c$. If $c < 1/2$, then $\bar{\gamma}_c(b, x_2) = (1/2, 1/2)$. Hence, $\mu_1(b, x_2) = 1/2 > \mu_1(x_1', x_2) = c$.

(7) If $x_2 \in [-\bar{x}, -b)$, then $(b, x_2) \in Z_{11}$. Hence, $\mu_1(b, x_2) = 1 \geq \mu_1(x_1', x_2)$ for any $\bar{\gamma}_c \in \bar{\Gamma}$.

Thus, strategy $\hat{\alpha}_1$ weakly dominates strategy α_1, which is a contradiction.

($\Delta([0, b])^* \subseteq A_i$) We have to consider the following scenarios depending on the media bias. First, we suppose that $0 < b \leq r/2$ (Case (i)). Without loss of generality, assume that $i = 1$, and fix $\alpha_1 \in \Delta([0, b])^*$ and $\alpha_1' \in \Delta(X)^*$ with $\alpha_1 \neq \alpha_1'$, arbitrarily. Note that $(x_1, l) \in Z_{11}$ for any $x_1 \in [0, b]$. That is, as long as we focus on the induced outcome of the news-reporting stage $\bar{\gamma}_c \in \bar{\Gamma}$, the opportunistic-type candidate 1 wins with certainty if candidate 2 is the ideological type. Hence, it is sufficient to compare the winning probabilities when candidate 2 is also the opportunistic-type.

Because $\alpha_1 \neq \alpha_1'$, there exists $x_1' \in S(\alpha_1)$ such that $\alpha_1(x_1') \neq \alpha_1'(x_1')$. If $\alpha_1(x_1') > \alpha_1'(x_1')$, then take $\alpha_2 = x_1'$ and $\bar{\gamma}_0 \in \bar{\Gamma}$. By construction, $\mu_1(x_1, x_1') = 1/2$ if $x_1 = x_1'$ and 0 otherwise. Hence:

$$\bar{U}_1(\alpha_1, \alpha_2, \bar{\gamma}_0) - \bar{U}_1(\alpha_1', \alpha_2, \bar{\gamma}_0) = \frac{1}{2}p(\alpha_1(x_1') - \alpha_1'(x_1')) > 0. \tag{B.7}$$

If $\alpha_1(x_1') < \alpha_1'(x_1')$, then take $\alpha_2 = x_1'$ and $\bar{\gamma}_1 \in \bar{\Gamma}$. By construction, $\mu_1(x_1, x_1') = 1/2$ if $x_1 = x_1'$ and 1 otherwise. Hence:

$$\begin{aligned}\bar{U}_1(\alpha_1, \alpha_2, \bar{\gamma}_1) - \bar{U}_1(\alpha_1', \alpha_2, \bar{\gamma}_1) &= \left(1 - \frac{1}{2}p\alpha_1(x_1')\right) - \left(1 - \frac{1}{2}p\alpha_1'(x_1')\right) \\ &= \frac{1}{2}p(\alpha_1'(x_1') - \alpha_1(x_1')) > 0. \end{aligned} \tag{B.8}$$

Thus, because α_1' is arbitrary, strategy α_1 is undominated with respect to $\bar{\Gamma}$; that is, $\alpha_1 \in A_1$ holds. Because α_1 is arbitrary, $\Delta([0, b])^* \subseteq A_1$ holds.

Second, we suppose that $r/2 < b \leq r$ (Case (ii)). By the similar argument as in Case (i), we can show that $\Delta([0, b])^* \subseteq A_1$. Next, we suppose that $i = 2$. Fix $\alpha_2 \in \Delta([0, b])^*$ and $\alpha_2' \in \Delta(X)^*$ with $\alpha_2 \neq \alpha_2'$, arbitrarily. Define $G(\alpha_2, \alpha_2') \equiv \{x_2 \in S(\alpha_2) | \alpha_2(x_2) \neq \alpha_2'(x_2)\}$.

Case (ii)-1: There exists $x_2' \in G(\alpha_2, \alpha_2') \cap [-r + 2b, b]$ such that $\alpha_2(x_2') < \alpha_2'(x_2')$.

Fix $\alpha_1 = x_2'$ and $\bar{\gamma}_0 \in \bar{\Gamma}$. In this scenario, $(r, x_2) \in Z_{22} \cup Z_{20} \cup Z_{21}$ for any $x_2 \in [0, b]$. Hence, $\bar{\gamma}_0(r, x_2) = (0, 1)$ for any $x_2 \in [0, b]$; that is, candidate 2 wins with certainty when candidate 1 is the ideological type. It is then sufficient to compare the winning probabilities when candidate 1 is opportunistic type. By construction:

$$\begin{aligned}\bar{U}_2(\alpha_1, \alpha_2, \bar{\gamma}_0) - \bar{U}_2(\alpha_1, \alpha_2', \bar{\gamma}_0) &= \left(1 - \frac{1}{2}p\alpha_2(x_2')\right) - \left(1 - \frac{1}{2}p\alpha_2'(x_2')\right) \\ &= \frac{1}{2}p(\alpha_2'(x_2') - \alpha_2(x_2')) > 0. \end{aligned} \tag{B.9}$$

Case (ii)-2: $G(\alpha_2, \alpha_2') \cap [-r + 2b, b] \neq \emptyset$ and $\alpha_2(x_2) > \alpha_2'(x_2)$ for any $x_2 \in G(\alpha_2, \alpha_2') \cap [-r + 2b, b]$.

In this scenario, there exists $x_2' \in G(\alpha_2, \alpha_2') \cap [0, -r+2b)$ such that $\alpha_2(x_2') < \alpha_2'(x_2')$; otherwise, either $\sum_{x_2 \in X} \alpha_2(x_2) > 1$ or $\sum_{x_2 \in X} \alpha_2'(x_2) < 1$ holds. Fix $\alpha_1 = x_2'$ and $\bar{\gamma}_0 \in \bar{\Gamma}$. Note that $\bar{\gamma}_0(r, x_2) = (0, 1)$ for any $x_2 \in [0, b]$. Hence, by the same argument used in Case (ii)-1, we can show that $\bar{U}_2(\alpha_1, \alpha_2, \bar{\gamma}_0) - \bar{U}_2(\alpha_1, \alpha_2', \bar{\gamma}_0) > 0$.

Case (ii)-3: $G(\alpha_2, \alpha_2') \cap [-r + 2b, b] = \emptyset$.

In this scenario, there exists $x_2' \in G(\alpha_2, \alpha_2') \cap [0, -r+2b)$. Furthermore, $\alpha(x_2) = \alpha_2'(x_2)$ for any $x_2 \in [-r + 2b, b]$. That is, candidate 2's winning probabilities from strategies α_2 and α_2' when candidate 1 is the ideological type are equivalent. It is then sufficient to compare the winning probabilities when candidate 1 is the opportunistic type. By the similar argument used in Case (i), we can show that there exists $\alpha_1 \in \Delta(X)^*$ and $\bar{\gamma}_c \in \bar{\Gamma}$ such that $\bar{U}_2(\alpha_1, \alpha_2, \bar{\gamma}_c) > \bar{U}_2(\alpha_1, \alpha_2', \bar{\gamma}_c)$. Because α_2' is arbitrarily, strategy α_2 is undominated with respect to $\bar{\Gamma}$; that is, $\alpha_2 \in A_2$. Because α_2 is arbitrarily, $\Delta([0, b])^* \subseteq A_2$.

Third, we suppose that $r < b < |l|$ (Case (iii)). By the similar argument used in Case (i), we can show that $\Delta([0, b])^* \subseteq A_1$. Next, we suppose that $i = 2$. Fix $\alpha_2 \in \Delta([0, b])^*$ and $\alpha_2' \in \Delta(X)^*$ with $\alpha_2 \neq \alpha_2'$, arbitrarily. Note that as long as $x_1 \in [0, b]$, any possible policy pair lies in regions $Z_0 \cup Z_{12} \cup Z_{21}$.

Case (iii)-1: $\alpha_2(r) \neq \alpha_2'(r)$.

If $\alpha_2(r) < \alpha_2'(r)$, then fix $\alpha_1 = r$ and $\bar{\gamma}_0 \in \bar{\Gamma}$. Hence:

$$\begin{aligned}\bar{U}_2(\alpha_1, \alpha_2, \bar{\gamma}_0) - \bar{U}_2(\alpha_1, \alpha_2', \bar{\gamma}_0) &= \left(1 - \frac{1}{2}\alpha_2(r)\right) - \left(1 - \frac{1}{2}\alpha_2'(r)\right) \\ &= \frac{1}{2}(\alpha_2'(r) - \alpha_2(r)) > 0. \end{aligned} \tag{B.10}$$

If $\alpha_2(r) > \alpha_2'(r)$, then fix $\alpha_1 = r$ and $\bar{\gamma}_1 \in \bar{\Gamma}$. Hence:

$$\bar{U}_2(\alpha_1, \alpha_2, \bar{\gamma}_1) - \bar{U}_2(\alpha_1, \alpha_2', \bar{\gamma}_1) = \frac{1}{2}(\alpha_2(r) - \alpha_2'(r)) > 0. \tag{B.11}$$

Case (iii)-2: $\alpha_2(r) = \alpha_2'(r)$.

Because $\alpha_2 \neq \alpha_2'$, there exists $x_2' \in S(\alpha_2)$ such that $\alpha_2(x_2') \neq \alpha_2'(x_2')$ and $x_2' \neq r$. If $\alpha_2(x_2') < \alpha_2'(x_2')$, then fix $\alpha_1 = x_2'$ and $\bar{\gamma}_0 \in \bar{\Gamma}$. Hence:

$$\begin{aligned}\bar{U}_2(\alpha_1, \alpha_2, \bar{\gamma}_0) - \bar{U}_2(\alpha_1, \alpha_2', \bar{\gamma}_0) &= \left(1 - \frac{1}{2}(p\alpha_2(x_2') + (1-p)\alpha_2(r))\right) - \left(1 - \frac{1}{2}(p\alpha_2'(x_2') + (1-p)\alpha_2'(r))\right) \\ &= \frac{1}{2}p(\alpha_2'(x_2') - \alpha_2(x_2')) > 0. \end{aligned} \tag{B.12}$$

If $\alpha_2(x_2') > \alpha_2'(x_2')$, then fix $\alpha_1 = x_2'$ and $\bar{\gamma}_1 \in \bar{\Gamma}$. Hence:

$$\begin{aligned}\bar{U}_2(\alpha_1, \alpha_2, \bar{\gamma}_1) - \bar{U}_2(\alpha_1, \alpha_2', \bar{\gamma}_1) &= \frac{1}{2}\left(p\alpha_2(x_2') + (1-p)\alpha_2(r)\right) - \frac{1}{2}(p\alpha_2'(x_2') + (1-p)\alpha_2'(r)) \\ &= \frac{1}{2}p(\alpha_2(x_2') - \alpha_2'(x_2')) > 0. \end{aligned} \tag{B.13}$$

Because α_2' is arbitrary, strategy α_2 is undominated with respect to $\bar{\Gamma}$; that is $\alpha_2 \in A_2$. Because α_2 is arbitrary, $\Delta([0, b])^* \subseteq A_2$.

Finally, we suppose that $b \geq |l|$ (Case (iv)). We can show that $\Delta([0,b])^* \subseteq A_2$ by the similar argument used in Case (iii). Next, we suppose that $i = 1$. Fix $\alpha_1 \in \Delta([0,b])^*$ and $\alpha_1' \in \Delta(X)^*$ with $\alpha_1 \neq \alpha_1'$, arbitrarily. Define $G(\alpha_1, \alpha_1') \equiv \{x_1 \in S(\alpha_1) | \alpha_1(x_1) \neq \alpha_1'(x_1)\}$.

Case (iv)-1: There exists $x_1' \in G(\alpha_1, \alpha_1') \cap [0, |l|)$ such that $\alpha_1(x_1') < \alpha_1'(x_1')$.

Fix $\alpha_2 = x_1'$ and $\bar{\gamma}_1 \in \bar{\Gamma}$. Note that because $(x_1, l) \in Z_{11} \cup \bar{Z}_{21}$ for any $x_1 \in [0,b]$, $\bar{\gamma}_1(x_1, l) = (1,0)$ holds for any $x_1 \in [0,b]$; that is, candidate 1 wins with certainty when candidate 2 is the ideological type. It is then sufficient to compare the winning probabilities when candidate 2 is the opportunistic type. Hence:

$$\begin{aligned}\bar{U}_1(\alpha_1, \alpha_2, \bar{\gamma}_1) - \bar{U}_1(\alpha_1', \alpha_2, \bar{\gamma}_1) &= \left(1 - \frac{1}{2}p\alpha_1(x_1')\right) - \left(1 - \frac{1}{2}p\alpha_1'(x_1')\right) \\ &= \frac{1}{2}p(\alpha_1'(x_1') - \alpha_1(x_1')) > 0. \qquad \text{(B.14)}\end{aligned}$$

Case (iv)-2: $G(\alpha_1, \alpha_1') \cap [0, |l|) \neq \emptyset$ and $\alpha_1(x_1) > \alpha_1'(x_1)$ for any $x_1 \in G(\alpha_1, \alpha_1') \cap [0, |l|)$.

In this scenario, there exists $x_1' \in G(\alpha_1, \alpha_1') \cap [|l|, b]$ such that $\alpha_1(x_1') < \alpha_1'(x_1')$; otherwise, either $\sum_{x_1 \in X} \alpha_1(x_1) > 1$ or $\sum_{x_1 \in X} \alpha_1'(x_1) < 1$ holds. Fix $\alpha_2 = x_1'$ and $\bar{\gamma}_1 \in \bar{\Gamma}$. Hence, $\bar{\gamma}_1(x_1, l) = (1,0)$ holds for any $x_1 \in [0,b]$. That is, we can show that $\bar{U}_1(\alpha_1, \alpha_2, \bar{\gamma}_1) - \bar{U}_1(\alpha_1', \alpha_2, \bar{\gamma}_1) > 0$ by the same argument used in Case (iv)-1.

Case (iv)-3: $G(\alpha_1, \alpha_1') \cap [0, |l|) = \emptyset$.

In this scenario, there exists $x_1' \in G(\alpha_1, \alpha_1') \cap [|l|, b]$. Note that because $G(\alpha_1, \alpha_1') \cap [0, |l|) = \emptyset$, $\alpha_1(x_1) = \alpha_1'(x_1)$ holds for any $x_1 \in [0, |l|)$. Then, $\sum_{x_1 \in [0,|l|)} \alpha_1(x_1) = \sum_{x_1 \in [0,|l|)} \alpha_1'(x_1)$. There are the following three cases to be checked. First, suppose that $\alpha_1(x_1') < \alpha_1'(x_1')$. Fix $\alpha_2 = x_1'$ and $\bar{\gamma}_1 \in \bar{\Gamma}$. Hence, $\bar{\gamma}_1(x_1, l) = (1,0)$ holds for any $x_1 \in [0,b]$. That is, we can show that $\bar{U}_1(\alpha_1, \alpha_2, \bar{\gamma}_1) - \bar{U}_1(\alpha_1', \alpha_2, \bar{\gamma}_1) > 0$ by the same argument used in Case (iv)-1. Second, suppose that $\alpha_1(x_1') > \alpha_1'(x_1')$ and $\alpha_1(|l|) < \alpha_1(|l|)$. Fix $\alpha_2 = |l|$ and $\bar{\gamma}_1 \in \bar{\Gamma}$. Note that $\bar{\gamma}_1(x_1, l) = (1,0)$ for any $x_1 \in [0,b]$. Hence, we can show that $\bar{U}_1(\alpha_1, \alpha_2, \bar{\gamma}_1) - \bar{U}_1(\alpha_1', \alpha_2, \bar{\gamma}_1) > 0$ by the similar argument used in Case (iv)-1. Finally, suppose that $\alpha_1(x_1') > \alpha_1'(x_1')$ and $\alpha_1(|l|) \geq \alpha_1'(|l|)$. Fix $\alpha_2 = x_1'$ and $\bar{\gamma}_0 \in \bar{\Gamma}$. Note that $\bar{\gamma}_0(|l|, l) = (1/2, 1/2)$, and $\bar{\gamma}_0(x_1, l) =$

$(0,1)$ for any $x_1 \in (|l|, b]$. Hence:

$$\begin{aligned}\bar{U}_1(\alpha_1, \alpha_2, \bar{\gamma}_0) - \bar{U}_1(\alpha_1', \alpha_2, \bar{\gamma}_0) &= \left(\frac{1}{2}p\alpha_1(x_1') + \frac{1}{2}(1-p)\alpha_1(|l|) + (1-p)\sum_{x_1 \in [0,|l|)} \alpha_1(x_1)\right) \\ &\quad - \left(\frac{1}{2}p\alpha_1'(x_1') + \frac{1}{2}(1-p)\alpha_1'(|l|) + (1-p)\sum_{x_1 \in [0,|l|)} \alpha_1'(x_1)\right) \\ &= \frac{1}{2}p(\alpha_1(x_1') - \alpha_1'(x_1')) + \frac{1}{2}(1-p)(\alpha_1(|l|) - \alpha_1'(|l|)) > 0.\end{aligned} \tag{B.15}$$

Because α_1' is arbitrarily, strategy α_1 is undominated with respect to $\bar{\Gamma}$; that is, $\alpha_1 \in A_1$. Because α_1 is arbitrary, $\Delta([0,b])^* \subseteq A_1$. ■

B.3 Robustness: Baseline Model

In this section, we discuss the robustness of the results by relaxing the assumptions of (i) single media outlet, (ii) asymmetry between the candidates, (iii) the tie-breaking rules and (iv) the non-strategicness of the ideological type. We demonstrate that the $(0,0)$ equilibrium is still fragile under relaxing assumptions (i), (iii) and (iv), which suggests that we can obtain the qualitatively same results in these scenarios. That is, assumptions (i), (iii) and (iv) are not essential to the results. On the other hand, assumption (ii) is crucial to the results in the sense that the symmetric candidates make the $(0,0)$ equilibrium more persistent. Hereafter, we use the USE as a solution concept, so the message space is modified: $M(z) = \{z, \phi\}$ for any $z \in Z$ where $m = \phi$ means suppression of the information.

B.3.1 Multiple media outlets

The assumption of single media outlet seems demanding, but crucial to the results of the baseline model. Imagine a scenario where there exist multiple media outlets whose preferences are opposing biased. Because the outlets have opposing-biased preferences, then the voter certainly learns the truth by observing both messages. In other words, if one outlet has an incentive to suppress information, then the other outlet definitely has an incentive to disclose it. Because the voter learns the true information, equilibrium outcomes are never distorted.[62] Thus, because ideologically different media outlets coexist in mature democracies, electoral outcomes seem little biased in

[62] This phenomenon is well-known in the literature of persuasion games. See, for example, Milgrom and Roberts (1986) and Lipman and Seppi (1995).

practice. However, this conjecture is based on an implicit assumption that both media outlets are sufficiently influential. In other words, the mechanism of mutual checking does not work well if the influence of one outlet dominates that of the other as demonstrated in the following example.[63]

Example 1. There exist outlets L and R whose preference bias is b_L and b_R with $b_L < 0 < b_R$ and $|b_L| > |l|/2$ and $b_R > r/2$, respectively. Each outlet correctly observes policy pair z, and then simultaneously sends messages m_L and m_R. However, the voter might not recognize the messages. Let s_j be the voter's observation from outlet j. We assume that the voter observes $s_R = m_R$ for certain, but he observes $s_L = m_L$ and ϕ with probabilities 2/5 and 3/5, respectively. That is, outlet R is more influential than outlet L in the sense that messages from outlet R is more likely to be recognized by the voter than those from outlet L. We assume that the voter cannot distinguish whether message $m_j = \phi$ is sent or message $m_j = z$ does not reach after observing $s_j = \phi$. Furthermore, we assume that $p = 9/10$. The remaining setup is identical to that of the baseline model.

Claim B.1 *There does not exist the* $(0, 0)$ *equilibrium in Example 1.*

Proof. Suppose, in contrast, that there exists the $(0, 0)$ equilibrium. Notice that if $s \equiv (s_L, s_R) \neq (\phi, \phi)$, then the voter observes the truth. Hence, the voter's posterior after observing $s = (\phi, \phi)$ is $\mathcal{P}^*(z|\phi, \phi) = 1$ if $z = (r, 0)$ and 0 otherwise. Hence, the voter's best response to observation $s = (\phi, \phi)$ is choosing candidate 2 for certain. As a result, candidate 2's equilibrium winning probability is 11/20. However, if candidate 2 deviates to $\alpha_2 = r$, then his winning probability is 59/100. That is, candidate 2 has an incentive to deviate from $\alpha_2^* = 0$, which is a contradiction. ■

As demonstrated, coexistence of ideologically different outlets could not prevent distortion. When one outlet is sufficiently more influential than the other, the direct and indirect distortion appear because of the same reason in the baseline scenario. Hence, the single-outlet model can be interpreted as a reduced form of a multiple-outlet model whose influence is unbalanced as discussed in Section 2.2.2. That is, the single outlet in the baseline model is a representative outlet in a country where the aggregate media coverage is not neutral, which is frequently observed even in democratic counties.

[63]See the companion paper Miura (2013) for the detailed analysis.

B.3.2 Asymmetry between the candidates

We have assumed that the candidates are asymmetric in the sense that the preferred policies of the ideological candidates differ. In order to consider the importance of the asymmetry, we first consider the model where the candidates are completely symmetric in the following sense; for $i \in \{1, 2\}$, if candidate i is the ideological type, then he proposes $x_i = r$ for certain. Except for this modification, the setup is identical to that in the baseline model. The result is as follows.

Proposition B.3 *Consider the manipulated news model with symmetric candidates. Then, there exists a* $(0, 0)$ *equilibrium if and only if* $b \notin (r/2, r)$.

Proof. (Necessity) Suppose, in contrast, that there exists the $(0, 0)$ equilibrium when $r/2 \leq b < r$. Because we focus on USE and the candidates are symmetric, the induced outcome of the news-reporting stage is $\bar{\gamma}_{1/2}$. That is, candidate 1's winning probability from $\alpha_1^* = 0$ is $1/2$. However, if candidate 1 deviates to strategy $\alpha_1 = b$, then his winning probability is $1 - p/2$. Because $p < 1$, candidate 1 has an incentive to deviate, which is a contradiction.

(Sufficiency) Suppose that $b \notin (r/2, r)$. By the same argument used in the proof of Theorem 1, we can show that there exists the $(0, 0)$ equilibrium supported by strategy β^* and off-the-equilibrium-path beliefs $\mathcal{P}^*$ given by (A.6), (A.7) and (A.8), respectively. ■

In contrast with the baseline model, the $(0, 0)$ equilibrium exists except for $b \in (r/2, r)$.[64] The persistence of the policy convergence result is a consequence of symmetric candidates. Because the candidates are symmetric, the front-runner never exists; that is, the voter is indifferent between the candidates under message $m = \phi$. Hence, the candidates do not have a sufficient self-mediatization incentive that is the main force breaking down the $(0, 0)$ equilibrium. In other words, the asymmetry between the candidates reinforces the self-mediatization incentives. Therefore, we can conclude that the asymmetric setup is essential for the results.

While the asymmetry between the candidates is essential, we emphasize that this requirement is mild in the sense that excluding the symmetric case is sufficient to derive the baseline results. In the baseline model, we have assumed the following two kinds of asymmetry between the candidates. The first is *asymmetry in distance* in the sense that the voter has a strict preference for the policy pair given by the ideological-type candidates. The second is *asymmetry in direction* in the sense that one candidate prefers a positive policy, but the other prefers a negative policy when they are

[64] Notice that there exist multiple equilibria as in the baseline model because the benefit from appealing to the voter is discounted.

of the ideological type. We hereafter show that it is unnecessary to distinguish the difference; that is, either one of these asymmetries is sufficient to generate strong self-mediatization incentives.

First, we consider the asymmetry in distance using the following one-sided setup. We assume that if candidate 2 is the ideological type, then he always proposes policy $x_2 = r'$ with $r' > r$. The remaining setup is identical to the baseline model. Similar to the baseline model, the $(0, 0)$ equilibrium is still fragile.

Proposition B.4 *Consider the manipulated news model with the candidates being only asymmetric in distance. Then, there exists the* $(0, 0)$ *equilibrium if and only if* $b \leq r/2$.

Proof. For the necessity, we suppose, in contrast, that there exists the $(0, 0)$ equilibrium when $b > r/2$. First, suppose that $r/2 < b \leq r$. In this scenario, candidate 2 is the front-runner, and then he has an incentive to deviate to $\alpha_2 = b$, which is a contradiction. Next, suppose that $b > r$. In this scenario, the induced outcome of the news-reporting stage is either $\bar{\gamma}_0$ or $\bar{\gamma}_1$. However, in each case, the front-runner has an incentive to deviate $\alpha_i = \varepsilon > 0$ where ε is sufficiently small. For the sufficiency, the construction used in the proof of Theorem 1 is still valid when $b \leq r/2$. ■

It is worth mentioning the difference in applicability between the baseline and one-sided setups. The baseline model is reasonable to describe a situation where the candidates and the outlet have an ideological conflict. Conversely, the one-sided setup is more appropriate to represent the situation where they have quantitative conflicts. For instance, they agree about a reduction of military expenditure, but disagree about its amount. The manipulated news model can apply to both scenarios and predicts the same distortion mechanism.

Next, we discuss the asymmetry in direction by considering the following setup. We assume that candidate 2 of the ideological type always proposes policy $x_2 = -r$. Except for this modification, the setup is identical to that in the baseline model. Note that the candidates are only asymmetric in direction. Again, the $(0, 0)$ equilibrium is fragile in this symmetric two-sided setup when the preference bias is not small.

Proposition B.5 *Consider the manipulated news model with the candidates being only asymmetric in direction. Then, there exists a* $(0, 0)$ *equilibrium if and only if* $b \leq r/2$.

Proof We can show the statement by the similar argument used in the proof of Proposition B.4. ■

In summary, difference between the policies proposed by the ideological candidates 1 and 2 are necessary for inducing the fragility of the $(0, 0)$ equilibrium. However, as long as the candidates are asymmetric in the above sense, the $(0,0)$ equilibrium becomes fragile. In other words, we do not require a particular structure of asymmetry, and then the $(0,0)$ equilibrium is generically fragile. Therefore, we can conclude that the assumption of asymmetric candidates is a mild requirement.

B.3.3 Tie-breaking rules

The tie-breaking rules specified in Requirement 2 seem crucial to the results. While the tie-breaking rule for the voter is well accepted in the literature, for the outlet it would seem more controversial. We have assumed that the outlet discloses the information whenever the proposed policies are convergent, but there is no strong justification for this behavior. However, if the outlet suppresses the information, even when the proposed policies are convergent, then the serious multiplicity of equilibria occurs such that any strategy $\alpha_i \in \Delta([0,b])^*$ can be an equilibrium strategy.

Although this multiplicity is serious, most of the equilibria are not robust with respect to a small perturbation in the outlet's behavior. Instead of assuming full disclosure, we thus assume that the outlet discloses the information about convergent policies with probability $\varepsilon \in (0,1]$. That is, the outlet that observes the convergent policy pairs randomizes disclosure and suppression.[65] For easy reference, we call this tie-breaking rule the *ε-randomization rule*, and the original the *disclosure rule*. We can show that even if the probability of disclosure ε is sufficiently small, then the set of equilibrium policy pairs under the ε-randomization rule is equivalent to that under the disclosure rule. Therefore, we can justify focusing on the equilibria satisfying the disclosure rule from the viewpoint of robustness.

Let us introduce additional notation, and then modify the USE as follows. With abuse of notation, let $\beta(z) = (t, 1-t)$ represent that the outlet who observes policy pair z sends messages $m = z$ and ϕ with probabilities t and $1-t$, respectively. We say that the outlet's strategy β satisfies the ε-randomization rule if $\beta(z) = (\varepsilon, 1-\varepsilon)$ for any $z \in Z_{00}$. We say that the outlet's strategy β^ε is *ε-simple* if $\beta^\varepsilon(z) = (1,0)$ for any $z \in Z_{11} \cup Z_{22} \cup Z_0 \backslash Z_{00}$, $(\varepsilon, 1-\varepsilon)$ for any $z \in Z_{00}$, and $(0,1)$ for any $z \in Z_{12} \cup Z_{21}$. Let B^ε be the set of the ε-simple strategy of the outlet. We say that PBE $(\alpha_1^*, \alpha_2^*, \beta^*, \gamma^*; \mathcal{P}^*)$ is an *ε-USE* if (i) $\beta^* \in B^\varepsilon$; and (ii) $(\alpha_1^*, \alpha_2^*) \in \Delta([0,b])^{*2}$. Notice that the disclosure rule is 1-randomization rule, and then USE is equivalent to 1-USE. Let $EP^\varepsilon \equiv \{(\alpha_1^*, \alpha_2^*) \in \Delta([0,b])^{*2} \mid (\alpha_1^*, \alpha_2^*) \text{ can be supported in an } \varepsilon\text{-USE}\}$ be the set of equilibrium strategies of the opportunistic-type candidates under the ε-randomization rule.

Proposition B.6 *Consider the manipulated news model. Then, $EP^\varepsilon = EP^1$ for any $\varepsilon \in (0,1]$.*

[65] Because the result of the election is indifferent for the outlet when the proposed policy is convergent, such randomization can be supported as one of the best responses for the outlet.

Proof. Without loss of generality, assume that $r < b < |l|$, and fix $\varepsilon \in (0, 1)$, arbitrarily.[66] First, we show that $EP^1 \subseteq EP^\varepsilon$. Take $(\alpha_1, \alpha_2) \in EP^1$, arbitrarily. That is, there exists β^1, γ^1, and $\mathcal{P}^1$ such that $(\alpha_1, \alpha_2, \beta^1, \gamma^1, \mathcal{P}^1)$ is a USE. By Lemma 2-(ii), the induced outcome of the news-reporting stage is either $\bar{\gamma}_1$ or $\bar{\gamma}_{1/2}$.

First, we assume that the induced outcome of the news-reporting stage is $\bar{\gamma}_1$ (Case (i)). Now, we show that given α_1, α_2 and β^ε, γ^1 is the voter's best response. Let $\mathcal{P}^\varepsilon$ be the voter's consistent posterior given α_1, α_2 and β^ε. If $m = z$, then it is obvious that $\gamma^1(z) = y^v(z)$ is optimal. Because $\gamma^1(\phi) = (1, 0)$:

$$\begin{aligned}
&\sum_{z \in Z} |x_1| \mathcal{P}^1(z|\phi) < \sum_{z \in Z} |x_2| \mathcal{P}^1(z|\phi). \\
\iff& \sum_{z \in Z_{12} \cup Z_{21}} |x_1| \Pr(z|\alpha_1, \alpha_2) < \sum_{z \in Z_{12} \cup Z_{21}} |x_2| \Pr(z|\alpha_1, \alpha_2). \\
\iff& \sum_{z \in Z_{12} \cup Z_{21}} |x_1| \Pr(z|\alpha_1, \alpha_2) + (1-\varepsilon) \sum_{z \in Z_{00}} |x_1| \Pr(z|\alpha_1, \alpha_2) \\
&\qquad < \sum_{z \in Z_{12} \cup Z_{21}} |x_2| \Pr(z|\alpha_1, \alpha_2) + (1-\varepsilon) \sum_{z \in Z_{00}} |x_2| \Pr(z|\alpha_1, \alpha_2). \\
\iff& \sum_{z \in Z} |x_1| \mathcal{P}^\varepsilon(z|\phi) < \sum_{z \in Z} |x_2| \mathcal{P}^\varepsilon(z|\phi).
\end{aligned} \tag{B.16}$$

Therefore, we can say that γ^1 is the voter's best response under the ε-randomization rule.

Next, we show that given $\alpha_j, \beta^\varepsilon$ and γ^1, α_i is the best response of candidate i. Notice that the candidates' winning probabilities from strategies α_1 and α_2 in the disclosure rule are:

$$\begin{aligned}
\bar{U}_1(\alpha_1, \alpha_2, \bar{\gamma}_1) &= 1 - \frac{1}{2} \sum_{z \in Z_{00}} \Pr(z|\alpha_1, \alpha_2, \theta_1 = O) \\
&= \sum_{z \in [0,b]^2 \setminus Z_{00}} \Pr(z|\alpha_1, \alpha_2, \theta_1 = O) + \frac{1}{2} \left(1 - \sum_{z \in [0,b]^2 \setminus Z_{00}} \Pr(z|\alpha_1, \alpha_2, \theta_1 = O) \right).
\end{aligned} \tag{B.17}$$

$$\bar{U}_2(\alpha_1, \alpha_2, \bar{\gamma}_1) = \frac{1}{2} \sum_{z \in Z_{00}} \Pr(z|\alpha_1, \alpha_2, \theta_2 = O). \tag{B.18}$$

[66] By the similar argument used in this case, we can show that this statement holds in other cases. The details are available from the author upon the request.

Because $(\alpha_,\alpha_2) \in EP^1$, the following conditions should hold. For any α_1', $\alpha_2' \in \Delta([0,b])^*$:

$$\sum_{z\in[0,b]^2\setminus Z_{00}} \Pr(z|\alpha_1,\alpha_2,\theta_1=O) \geq \sum_{z\in[0,b]^2\setminus Z_{00}} \Pr(z|\alpha_1',\alpha_2,\theta_1=O). \tag{B.19}$$

$$\sum_{z\in Z_{00}} \Pr(z|\alpha_1,\alpha_2,\theta_2=O) \geq \sum_{z\in Z_{00}} \Pr(z|\alpha_1,\alpha_2',\theta_2=O). \tag{B.20}$$

Likewise, the candidates' winning probabilities under the ε-randomization rule are as follows:

$$\bar{U}_1(\alpha_1,\alpha_2,\bar{\gamma}_1^\varepsilon) = \sum_{z\in[0,b]^2\setminus Z_{00}} \Pr(z|\alpha_1,\alpha_2,\theta_1=O) + \left(1-\frac{\varepsilon}{2}\right)\left(1-\sum_{z\in[0,b]^2\setminus Z_{00}} \Pr(z|\alpha_1,\alpha_2,\theta_1=O)\right), \tag{B.21}$$

$$\bar{U}_2(\alpha_1,\alpha_2,\bar{\gamma}_1^\varepsilon) = \frac{\varepsilon}{2}\sum_{z\in Z_{00}} \Pr(z|\alpha_1,\alpha_2,\theta_2=O), \tag{B.22}$$

where $\bar{\gamma}_1^\varepsilon$ is the induced outcome of the news-reporting strategy under the ε-randomization rule. By (B.19) and $(B.20)$, we can say that α_i is a best response to α_j, β^ε and γ^1. Therefore, $(\alpha_1,\alpha_2,\beta^\varepsilon,\gamma^1;\mathcal{P}^\varepsilon)$ is an ε-USE; that is, $(\alpha_1,\alpha_2) \in EP^\varepsilon$.

Second, we suppose the induced outcome of the news-reporting stage is $\bar{\gamma}_{1/2}$ (Case (ii)). By the similar argument as in Case (i), we can show that γ^1 is the voter's best response given α_1,α_2 and β^ε. Hence, the candidates' winning probabilities under the ε-randomization rule are $\bar{U}_1(\alpha_1,\alpha_2,\bar{\gamma}_{1/2}^\varepsilon) = 1-p/2$ and $\bar{U}_2(\alpha_1,\alpha_2,\bar{\gamma}_{1/2}^\varepsilon) = 1/2$. That is, each candidate has no incentive to deviate from strategy α_i. Hence, $(\alpha_1,\alpha_2,\beta^\varepsilon,\gamma^1;\mathcal{P}^\varepsilon)$ is an ε-USE, which means that $(\alpha_1,\alpha_2) \in EP^\varepsilon$. Because (α_1,α_2) is arbitrary, we can conclude that $EP^1 \subseteq EP^\varepsilon$. By the similar argument, we can also show that $EP^\varepsilon \subseteq EP^1$.[67] Therefore, $EP^\varepsilon = EP^1$ holds for any $\varepsilon \in (0,1]$. ■

B.3.4 Fully strategic candidates

One of the most important factors in deriving the distortion mechanism is the voter's uncertainty about how the candidates behave. In other words, the nonstrategicness of the ideological type is irrelevant to the results. In the baseline model, we have assumed that there are two types of candidates, one of which is nonstrategic. In this subsection, we instead assume that the candidates are fully rational and office-motivated, but the candidates face uncertainty about the voter's preference. We then show the fragility of policy convergence in this new setup; that is, the nonstrategic ideological candidates is not essential.

[67] This statement does not hold if $\varepsilon = 0$. Furthermore, we can show that candidate 2 cannot be the front-runner even under the ε-randomization rule by the similar argument used in the proof of Lemma 2.

The baseline model is modified as follows. Let $d \in \{l, 0, r\}$ be the voter's ideal policy. We assume that ideal policy d is private information of the voter and the outlet; that is, the candidates do not know it. However, we do not assume common prior on d. Instead, we represent the players' beliefs by a type space of Harsanyi (1967-68) denoted by $\mathcal{T} \equiv (T_1, T_2, T_o, T_v; \lambda_1, \lambda_2, \lambda_o, \lambda_v)$. For any $i \in \{1, 2, o, v\}$, player i's type t_i is an element of measurable set T_i, and player i's belief is represented by a measurable function $\lambda_i : T_i \to \Delta(T_{-i})$.

We assume that the Harsanyi's type space $\mathcal{T}$ satisfies the following properties. For $i \in \{o, v\}$, let $d_i(t_i) \in \{l, 0, r\}$ be the ideal policy of the voter when player i is type t_i, and let $T_i^j \equiv \{t_i \in T_i \mid d_i(t_i) = j\}$ be the set of player i's types who believe that the voter's ideal policy is $j \in \{l, 0, r\}$. That is, $T_i = T_i^l \cup T_i^0 \cup T_i^l$ holds. Because both the voter and the outlet know the voter's true ideal policy for certain, the following condition should be satisfied.

Assumption B.1 *For any $(t_o, t_v) \in T_o \times T_v$, $d_o(t_o) = d_v(t_v)$ holds.*

We then assume that the candidates' beliefs over ideal policy d satisfy the following properties.

Assumption B.2

(i) For any $t_1 \in T_1$, either exactly one of the following condition holds: (1) $\int_{t_{-1}:t_v \in T_v^0} \lambda_1(t_{-1}|t_1)dt_{-1} = 1$ or (2) $\int_{t_{-1}:t_v \in T_v^r} \lambda_1(t_{-1}|t_1)dt_{-1} = 1$.

(ii) For any $t_2 \in T_2$, either exactly one of the following condition holds: (1) $\int_{t_{-2}:t_v \in T_v^0} \lambda_2(t_{-2}|t_2)dt_{-2} = 1$ or (2) $\int_{t_{-2}:t_v \in T_v^l} \lambda_2(t_{-2}|t_2)dt_{-2} = 1$.

In other words, we assume that candidate 1 (resp. 2) believes either (i) $d = 0$ for certain, or (ii) $d = r$ (resp. l) for certain. We refer to the former as *moderate type* (M), and the latter as *extreme type* (E). For candidate $i \in \{1, 2\}$, let $f_i(t_i) \in \{M, E\}$ represent that candidate with type t_i is whether the moderate or the extremist. For any $i \in \{1, 2\}$ and $j \in \{M, E\}$, define $T_i^j \equiv \{t_i^j \in T_i | f_i(t_i) = j\}$, and then $T_i = T_i^M \cup T_i^E$ holds. Finally we assume the following properties.

Assumption B.3

(i) For any $t_v \in T_v$ and $i \in \{1, 2\}$, $0 < \int_{t_{-v}:t_i \in T_i^M} \lambda_v(t_{-v}|t_v)dt_{-v} < 1$.

(ii) There exists $t_2 \in T_2^M$ such that $\int_{t_{-2}:t_1 \in T_1^M} \lambda_2(t_{-2}|t_2)dt_{-2} > 0$.

The first condition means that any type of the voter cannot pin down the type of the candidates. This condition is associated with the voter's uncertainty to the candidates' types in the baseline model. The second condition excludes the scenario where any type of candidate 2 certainly believes that candidate 1 is the extremist.

The candidates' and the outlet's preferences are still given by (1) and (3), respectively. The voter's von Neumann–Morgenstern utility function $u : Z \times Y \times T_v \to \mathbb{R}$ is defined as follows:

$$v(z, y, t_v) \equiv \begin{cases} -|x_1 - d_v(t_v)| & \text{if } y = y_1, \\ -|x_2 - d_v(t_v)| & \text{if } y = y_2. \end{cases} \tag{B.23}$$

The timing of the game is identical to that of the baseline model except that there is no nature's move. Then, the players' strategies and beliefs are defines as follows. Let $\alpha_i : T_i \to \Delta(Z)^*$ be candidate i's strategy, $\beta : T_o \times Z \to M$ be the outlet's strategy and $\gamma : T_v \times M \to \Delta(Y)$ be the voter's strategy.[68] Let $\mathcal{P} : T_v \times M \to \Delta(Z)$ be the voter's posterior belief over the policy pair space Z. Because of this modification, we adopt PBE with restrictions that (i) the voter's strategy is in set Γ, and (ii) the outlet's strategy is simple as a solution concept. Except for these modifications, this new setup is identical to that of the baseline model.

As a benchmark, we briefly discuss the situation where there is no media manipulation. Because the voter correctly observes policy pair z, the candidates directly appeal to the voter. That is, there exists a PBE where the candidates' strategies are the following:[69]

$$\alpha_1^*(t_1) = \begin{cases} 0 & \text{if } t_1 \in T_1^M, \\ r & \text{if } t_1 \in T_1^E; \end{cases} \tag{B.24}$$

$$\alpha_2^*(t_2) = \begin{cases} 0 & \text{if } t_2 \in T_2^M, \\ l & \text{if } t_2 \in T_2^E. \end{cases} \tag{B.25}$$

In this equilibrium, each candidate directly appeals to the voter upon his belief. Thus, the situation where the voter's ideal policy is 0 is associated with the baseline model; that is, from the perspective of the voter with type $t_v \in T_v^0$, candidate 1 (resp. 2) proposes policy $x_1 = 0$ (resp. $x_2 = 0$) with probability $p_1(t_v)$ (resp. $p_2(t_v)$) and policy $x_1 = r$ (resp. $x_2 = l$) with probability $1 - p_1(t_v)$ (resp. $1 - p_2(t_v)$) where $p_i(t_v) \equiv \int_{t_{-v}: t_i \in T_i^M} \lambda_v(t_{-v}|t_v) dt_{-v}$ for $i \in \{1, 2\}$. Hence, the moderate and the extremist are associated with the opportunistic-type and the ideological-type candidates in the baseline model, respectively. We call this equilibrium the *direct-appealing*

[68] The outlet is restricted to the simple strategies.
[69] The details are available from the author upon the request.

equilibrium, and adopt it as a reference point instead of the $(0,0)$ equilibrium in the baseline model.

Now, we move back to the scenario where the outlet behaves strategically. Because of the self-mediatization incentive of the candidates, the direct-appealing equilibrium brakes down as the baseline model.

Proposition B.7 *Consider the manipulated news model with fully rational candidates. Then, there exists the direct-appealing equilibrium if and only if $b \le r/2$.*

Proof. (Necessity) Suppose, in contrast, that there exists the direct-appealing equilibrium $(\alpha_1^*, \alpha_2^*, \beta^*, \gamma^*; \mathcal{P}^*)$ when $b > r/2$. Notice that $Z(\alpha_1^*, \alpha_2^*) \equiv \{(\alpha_1^*(t_1), \alpha_2^*(t_2)) | (t_1, t_2) \in T_1 \times T_2\} = \{(0,0), (0,l), (r,0), (r,l)\}$, and any type of the voter believes that it is the set of possible policy pairs on the equilibrium path by Assumption B.3-(i). If $t_v \in T_v^0$, then only policy pair $z = (r,0)$ exists in the disagreement region. That is, candidate 2 is the front-runner for any $t_v \in T_v^0$. By Assumptions B.2 and B.3-(ii), there exists type $t_2' \in T_2^M$ who believes that (i) the voter's ideal policy is 0 for certain, and (ii) candidate 1 is moderate with positive probability $q(t_2') \equiv \int_{t_{-2}: t_1 \in T_1^M} \lambda_2(t_{-2}|t_2') dt_{-2}$. Hence, the winning probability of type t_2' from $\alpha_2^*(t_2') = 0$ is $1 - q(t_2')/2$. However, if he proposes policy $x_2 = \varepsilon > 0$ where ε is small enough, then his winning probability is 1. That is, candidate 2 of type t_2' has an incentive to deviate, which is a contradiction.

(Sufficiency) Suppose that $b \le r/2$, and we show that the following is a PBE: α_1^* and α_2^* are given by (B.24) and (B.25), respectively:

$$\beta^*(t_o, z) = \begin{cases} \phi & \text{if } [t_o \in T_o^0 \cup T_o^l \text{ and } z \in \bar{Z}_{12} \cup Z_{21}] \text{ or } [t_o \in T_o^r \text{ and } z \in Z_{12} \cup \bar{Z}_{21}], \\ z & \text{otherwise;} \end{cases}$$

$$\gamma^*(t_v, m) = \begin{cases} y^v(z) & \text{if } m = z, \\ (1,0) & \text{if } m = \phi \text{ and } t_v \in T_v^r, \\ (1/2, 1/2) & \text{if } m = \phi \text{ and } t_v \in T_v^0, \\ (0,1) & \text{if } m = \phi \text{ and } t_v \in T_v^l, \end{cases} \tag{B.26}$$

$$S(\mathcal{P}^*(\cdot|t_v, \phi)) \subseteq \begin{cases} \{(b,b)\} & \text{if } t_v \in T_v^0, \\ \{(r,0)\} & \text{if } t_v \in T_v^r, \\ Z_{21} & \text{if } t_v \in T_v^l. \end{cases}$$

It is obvious that $\mathcal{P}^*$ is consistent with Bayes' rule given α_1^*, α_2^* and β^* and γ^* is optimal to the voter given $\mathcal{P}^*$. Also, given γ^*, it is obvious that β^* is optimal to the outlet. Thus, it is sufficient to show that α_1^* and α_2^* are optimal for any type of the candidates. First, consider candidate 1's behavior. For any type $t_1 \in T_1^M$, the winning probability from $\alpha_1^*(t_1) = 0$ is $1 - q_1(t_1)/2$ where

$q(t_1) \equiv \int_{t_{-1}:t_2 \in T_2^M} \lambda_1(t_{-1}|t_1)dt_{-1}$. Notice that there is no front-runner in this scenario. Hence, for any $x_1 \in X$, $\mu_1(x_1, 0) \leq 1/2$ and $\mu_1(x_1, l) \leq 1$. That is, any type $t_1 \in T_1^M$ of candidate 1 has no incentive to deviate. For any $t_1 \in T_1^E$, the winning probability from $\alpha_1^*(t_1) = r$ is 1; that is, he has no incentive to deviate. Therefore, α_1^* is optimal to candidate 1. Likewise, we can show that α_2^* is optimal to candidate 2. ■

B.4 Supplementary Materials for No Competition Model

B.4.1 Set of equilibria

The characterization of the equilibrium set for the remaining cases is as follows.

Proposition B.8 *Consider the no competition model.*

(i) If $0 < b \leq r/2$*, then* $D_{nc}(\alpha_1, p) = [p(1-p)\hat{x}_1, p\hat{x}_1 + p(1-p)b]$.

(ii) If $r/2 < b < r$*, then:*

$$D_{nc}(\alpha_1, p) = \begin{cases} [p(1-p)\hat{x}_1, p\hat{x}_1 + p(1-p)b] & \text{if } \hat{x}_1 \neq 0, \\ (0, p(1-p)b] & \text{otherwise.} \end{cases} \tag{B.27}$$

(iii) If $b \geq |l|$*, then:*

$$D_{nc}(\alpha_1, p) = \begin{cases} [p(1-p)\rho(\alpha_1), p\hat{x}_1 + p(1-p)r] & \text{if } \hat{x}_1 \neq 0, \\ (0, p(1-p)r] & \text{otherwise,} \end{cases} \tag{B.28}$$

where $\rho(\alpha_1) \equiv \sum_{x_1 \in [0,|l|)} x_1 \alpha_1(x)_1 + |l| \sum_{x_1 \in [|l|,b]} \alpha_1(x_1)$.

Proof. (i) Suppose that $0 < b \leq r/2$. Without loss of generality, we can focus on the equilibrium where candidate 2 adopts $\alpha_2 = x_2 \in [0, b]$ with inducing winning probability 1, as an analogy of the proof of Proposition 3 in the body of the paper. Notice that the information is suppressed if and only if both candidates are the opportunistic type. Hence, the voter's response to the suppressed message depends on $\hat{x}_1$ and x_2. If $x_2 \leq \hat{x}_1$, then the distortion is $d(\alpha_1, x_2) = px_2 + p(1-p)\hat{x}_1$. Otherwise, $d(\alpha_1, x_2) = p\hat{x}_1 + p(1-p)x_2$. Because the distortion level is monotonic in x_2 as in Figure B.1, its set is characterized as in the statement.

(ii) Suppose that $r/2 < b < r$. In this scenario, policy pair $x = (r, x_2)$ is suppressed if and only if $x_2 \in [0, -r + 2b)$. Hence, there are the following cases to be checked.

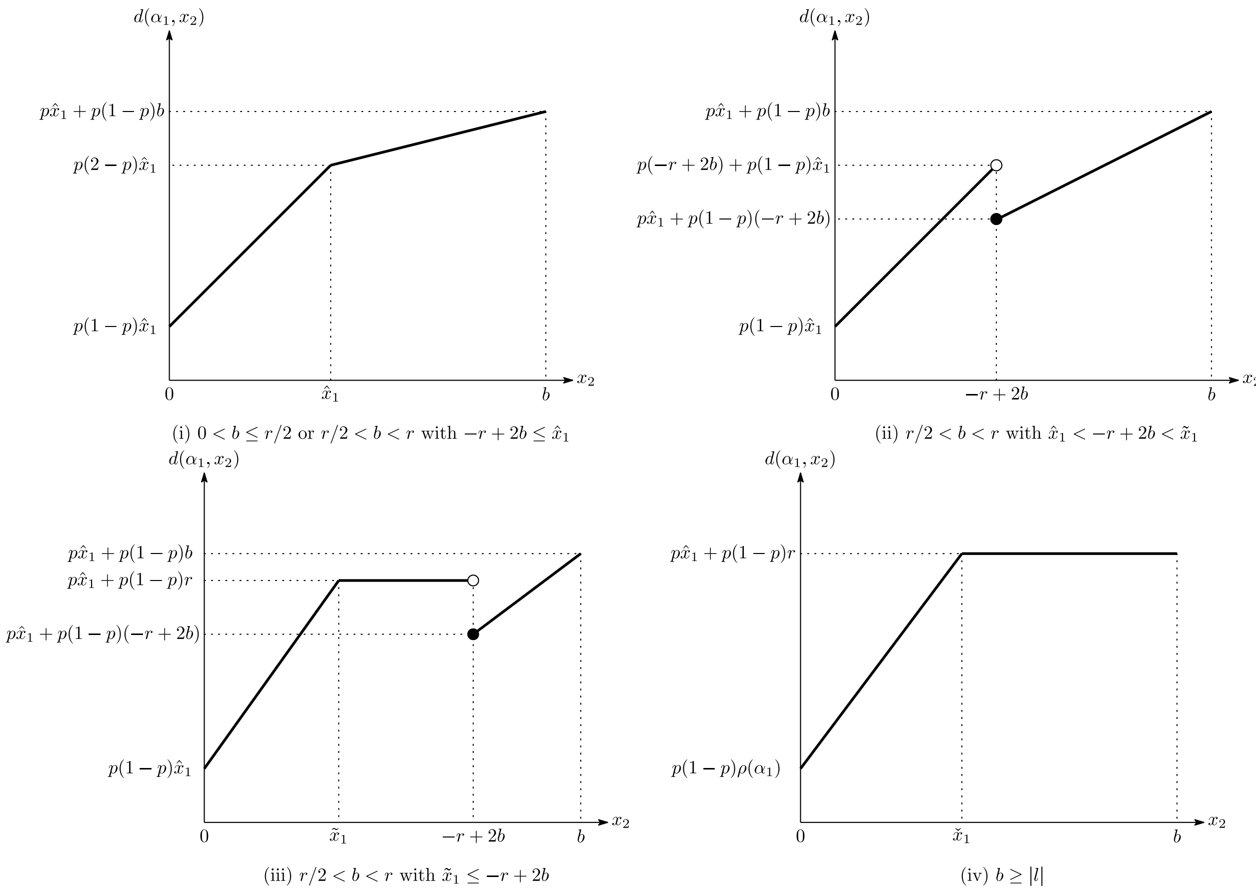

Figure B.1: Distortion level in the no competition model

Case (a): $-r + 2b \leq \hat{x}_1$.

In this scenario, $d(\alpha_1, x_2)$ is equivalent to Case (i).

Case (b): $\hat{x}_1 < -r + 2b < \tilde{x}_1$.

In this scenario, the voter's response to the suppressed message is $\gamma^*(\bar{Z}_{12} \cup \bar{Z}_{21}) = (1, 0)$ (resp. $(0, 1)$) if $x_2 < -r+2b$ (resp. $x_2 > -r+2b$). Hence, the distortion is $d(\alpha_1, x_2) = px_2 + p(1-p)\hat{x}_1$ if $x_2 \in [0, -r + 2b)$. Otherwise, $d(\alpha_1, x_2) = p\hat{x}_1 + p(1 - p)x_2$.

Case (c): $\tilde{x}_1 \leq -r + 2b$.

In this scenario, the voter's response to the suppressed message is $\gamma^*(\bar{Z}_{12} \cup \bar{Z}_{21}) = (1, 0)$ (resp. $(0, 1)$) if $x_2 < \tilde{x}_1$ (resp. $x_2 > \tilde{x}_1$). Furthermore, if $x_2 \in (\tilde{x}_1, -r + 2b)$, then policy pair $x = (r, x_2)$ is in the disagreement regions. Otherwise, it lies in the agreement regions. Thus, the degree of distortion is characterized as follows:

$$d(\alpha_1, x_2) = \begin{cases} px_2 + p(1-p)\hat{x}_1 & \text{if } x_2 \in [0, \tilde{x}_1], \\ p\hat{x}_1 + p(1-p)r & \text{if } x_2 \in (\tilde{x}_1, -r + 2b), \\ p\hat{x}_1 + p(1-p)x_2 & \text{otherwise.} \end{cases} \tag{B.29}$$

In each case, the distortion is monotonic in x_2 as in Figure B.1, the characterization of its set is as in the statement.

(iii) Suppose that $b \geq |l|$. Notice that, in this scenario, policy $x = (x_1, l)$ is suppressed if and only if $x_1 \in [|l|, b]$. That is, the voter's expected utility from actions y_1 and y_2 conditional on $m = \bar{Z}_{12} \cup \bar{Z}_{21}$ is $p\tilde{x}_1 + p(1-p) \sum_{x_1 \in [|l|, b]} x_1 \alpha_1(x_1)$ and $px_2 + p(1-p)|l| \sum_{x_1 \in [|l|, b]} \alpha_1(x_1)$, respectively. Therefore, the voter's best response to the suppressed message is $\gamma^*(\bar{Z}_{12} \cup \bar{Z}_{21}) = (1, 0)$ (resp. $(0, 1)$) if $x_2 < \check{x}_1$ (resp. $x_2 > \check{x}_1$), where $\check{x}_1 \equiv \tilde{x}_1 + (1 - p)\{\sum_{x_1 \in [|l|, b]} x_1 \alpha_1(x_1) - |l| \sum_{x_1 \in [|l|, b]} \alpha_1(x_1)\}$. Therefore, the distortion is $d(\alpha_1, x_2) = px_2 + p(1 - p)\rho(\alpha_1)$ if $x_2 \leq \check{x}_1$. Otherwise, $d(\alpha_1, x_2) = p\hat{x}_1 + p(1 - p)r$. Because the distortion is monotonic in x_2 as shown in Figure B.1, its set is characterized as in the statement. ■

The following remarks should be mentioned. First, the first-best outcome for the voter is, at least approximately, attainable in equilibrium by taking $\alpha_1 = 0$. That is, compared with the baseline model in which the $(0, 0)$ equilibrium does not exist, the distortion could be underestimated. This misspecification comes from the fact that candidate 1's incentive conditions can be ignored, as pointed out in the body of the paper. Second, if the bias is sufficiently small, i.e., $b \leq r/2$, then the misspecification never occurs. Because the bias is sufficiently small, the information is suppressed only when both candidates are the opportunistic type. Hence, in this scenario, if both candidates adopt an identical strategy, then it constructs a no-front-runner equilibrium, as pointed out above. In other words, candidate 2's strategies that can be supported in equilibrium are never restricted by candidate 1's incentive compatibility condition. Therefore, the characterization of the equilibrium set is irrelevant to whether candidate 1's incentive conditions are omitted. Finally, if the bias is sufficiently large, i.e., $b \geq |l|$, then the distortion could be overestimated, which also comes from ignoring candidate 1's incentive conditions. In the baseline model, proposing policy $x_1 \in [|l|, b]$ is not incentive compatible, which determines the upper bound of the distortion. However, x_1 can be greater than $|l|$ because we can ignore candidate 1's incentive conditions. As a result, the distortion could exceed the upper bound of the baseline model.

B.4.2 Robustness

In the no competition model discussed in the body of the paper, we assume that α_1 is exogenously fixed, and demonstrate that the lower bound can be underestimated. In this section, we show that this underestimation can be observable even if α_2 is exogenously fixed. We assume that $\alpha_2 \in \Delta^*([0, b])$ is exogenously given, and let $\hat{x}_2 \equiv \sum_{x_2 \in X} x_2 \alpha_2(x_2)$ be the expected policy of candidate 2. Except for this modification, the setup is equivalent to that of the no competition model in the body of the paper. As shown in the following proposition, the first-best outcome is approximately attainable.

Proposition B.9 *Consider the no competition model with fixed α_2 and $r \leq b < |l|$.*

(i) If $\alpha_2(\varepsilon/2) = \alpha_2(3\varepsilon/2) = 1/2$, then there exists a USE in which $\alpha_1^* = 3\varepsilon/2$.

(ii) For any $\delta > 0$, there exist an equilibrium e such that $d(\alpha_1^*, \alpha_2; e) < \delta$.

Proof. (i) Notice that, by taking $\varepsilon > 0$ sufficiently small, $\hat{x}_2 = \varepsilon < p(3\varepsilon/2) + (1-p)r$ holds. Thus, the induced outcome of the news-reporting stage is $\bar{\gamma}_0$. Now, we construct the desired equilibrium in which the outlet's strategy and the voter's off-the-equilibrium-path beliefs are given by (A.54) and (A.55) in the body of the paper, respectively. Thus, the optimality of β^* is shown by the same argument. It is then sufficient to show the optimality of α_1^*. Notice that $\bar{U}_1(\alpha_1^*, \alpha_2, \bar{\gamma}_0) = 1 - 3p/4$. By construction, if candidate 1 deviates to $\alpha_1' = x_1 \in [0, b]$, then his winning probability is $\bar{U}_1(\alpha_1', \alpha_2, \bar{\gamma}_0) \leq 1 - 3p/4$. Thus, he has no incentive to deviate, and then it is a USE.

(ii) Fix $\delta > 0$, arbitrarily. The degree of distortion of equilibrium constructed in (i) is:

$$d(\alpha_1^*, \alpha_2) = p^2 \hat{x}_2 + p(1-p)\hat{x}_2 + p(1-p)\left(\frac{3\varepsilon}{2}\right) = p(4-3p)\varepsilon. \tag{B.30}$$

Therefore, by taking $\varepsilon < \delta/(p(4-3p))$, $d(\alpha_1^*, \alpha_2) < \delta$ holds. ■

B.5 Supplementary Materials for Nonstrategic Outlet Model

B.5.1 Omitted results

As a corollary of Proposition 4 in the body of the paper, the misspecification of the distortion in the nonstrategic model is summarized as follows. To simplify the representation, let $\underline{D}$ and $\overline{D}$ represent the infimum and supremum of the baseline model characterized by Theorem 2 in the body of the paper, respectively.

Corollary B.1 *Consider the nonstrategic outlet model.*

(i) Suppose that $0 < b \leq (1-p)qr/(2-p)$. *Then,* $\underline{D} < \overline{D} \leq d(q)$.

(ii) Suppose that $(1-p)qr/(2-p) < b \leq r/2$. *Then,* $\underline{D} < d(q) < \overline{D}$.

(iii) Suppose that $r/2 < b < r$.

- *(a) If* $q \leq r/(1+r)$, *then* $d(q) < \underline{D} < \overline{D}$.
- *(b) If* $q > r/(1+r)$ *and* $r/2 < b \leq (1+r)q/2$, *then* $\underline{D} \leq d(q) < \overline{D}$.
- *(c) If* $q > r/(1+r)$ *and* $(1+r)q/2 < b < r$, *then* $d(q) < \underline{D} < \overline{D}$.

(iv) Suppose that $b \geq r$. *Then,* $d(q) < \underline{D} < \overline{D}$.

Proof. (i) and (ii) are obvious from the fact that:

$$p(1-p)qr \geq p(2-p)r \Longleftrightarrow b < \frac{(1-p)q}{2-p}r < \frac{1}{2}r. \tag{B.31}$$

(iii) The statements are obvious from the facts that:

$$p(1-p)qr \geq p(1-p)(-r+2b) \Longleftrightarrow b \leq \frac{1}{2}(1+r)q; \tag{B.32}$$

$$\frac{1}{2}(1+r)q \leq \frac{1}{2}r \Longleftrightarrow q \leq \frac{r}{1+r}. \tag{B.33}$$

(iv) The statement is obvious because $q < 1$. ■

B.5.2 Robustness

In this subsection, we consider the modified model in which the manipulation probability varies depending on the policy pairs. We assume that the outlet sends $m = \phi$ with probability $q_1 \in (0, 1)$ (resp. $q_2 \in (0, 1)$), and sends $m = z$ with the remaining probability if policy pair z is in the agreement regions (resp. disagreement regions). Define $\bar{q}_1 \equiv 1 - 2(1 - q_2)/(1 - p)$, and we put the following assumptions on the manipulation probabilities.

Assumption B.4

(i) $(pq_2 + (1 - p)q_1)r < q_1|l|$.

(ii) $q_1 \leq q_2$.

(iii) $q_1 \neq 2q_2 - 1,\ \bar{q}_1$.

The first assumption guarantees that if there exists a $(0, 0)$ equilibrium, then the front-runner is candidate 1, which is satisfied when r and $|l|$ diverge sufficiently. The second one means that the outlet suppresses the information more frequently when the policy pairs are in the disagreement region. The last one is for technical reasons, but it generically holds. Except for this modification, the setup is identical to those in the nonstrategic outlet model discussed in the body of the paper. For ease of reference, we call this model the *modified nonstrategic outlet model.* Furthermore, to clarify the argument, we focus on the scenario in which $r \leq b < |l|$. First, we show the following useful lemmas.

Lemma B.2 *Consider the modified nonstrategic outlet model, and suppose that $r \leq b < |l|$. If there exists an equilibrium in which the front-runner is candidate 1, then $x_1 < x_2$ holds for any $x_1 \in S(\alpha_1^*)$ and $x_2 \in S(\alpha_2^*)$.*

Proof. Suppose, in contrast, that there exist $x_1' \in S(\alpha_1^*)$ and $x_2' \in S(\alpha_2^*)$ such that $x_1' \geq x_2'$. Because $x_1' \in S(\alpha_1^*)$, candidate 1's equilibrium payoff is:

$$\bar{U}_1(\alpha_1^*, \alpha_2^*, \bar{\gamma}_1) = p\left(\frac{1}{2}(1 + q_1)\alpha_2^*(x_1') + \sum_{x_2 > x_1'} \alpha_2^*(x_2) + q_2 \sum_{x_2 < x_1'} \alpha_2^*(x_2)\right) + (1 - p). \tag{B.34}$$

Now, suppose that candidate 1 deviates to $\alpha_1 = x_1''$ where $0 < x_1'' < \min S(\alpha_2^*)\backslash\{0\}$, and then

$\bar{U}_1(x_1'', \alpha_2^*, \bar{\gamma}_1) = p(q_2\alpha_2^*(0) + (1 - \alpha_2^*(0))) + (1 - p)$. However:

$$\bar{U}_1(x_1'', \alpha_2^*, \bar{\gamma}_1) - \bar{U}_1(\alpha_1^*, \alpha_2^*, \bar{\gamma}_1) = p\left(\frac{1}{2}(1 - q_1)\alpha_2^*(x_1') + (1 - q_2)\sum_{0<x_2<x_1'} \alpha_2^*(x_2)\right) > 0, \quad \text{(B.35)}$$

where the inequality comes from there exists $x_2' \in S(\alpha_2^*)$ such that $x_2' \leq x_1'$. That is, candidate 1 has an incentive to deviate, which is a contradiction. ■

Lemma B.3 *Consider the modified nonstrategic outlet model, and suppose that $r \leq b < |l|$. If there exists an equilibrium in which the front-runner is candidate 2, then $x_2 < \min\{r, \min(S(\alpha_1^*)\backslash\{0\})\}$ holds for any $x_2 \in S(\alpha_2^*)$.*

Proof. Suppose, in contrast, that either (i) there exists $x_2' \in S(\alpha_2^*)$ such that $x_2' \geq r$, or (ii) there exist $x_1'' \in S(\alpha_1^*)\backslash\{0\}$ and $x_2'' \in S(\alpha_2^*)$ such that $x_2'' \geq x_1''$. First, we consider case (i), and suppose that $x_2' > r$. Because $x_2' \in S(\alpha_2^*)$:

$$\bar{U}_2(\alpha_1^*, \alpha_2^*, \bar{\gamma}_0) = p\left(\frac{1}{2}(1 + q_1)\alpha_1^*(x_2') + \sum_{x_1>x_2'} \alpha_1^*(x_1) + q_2\sum_{x_1<x_2'} \alpha_1^*(x_1)\right) + (1 - p)q_2. \quad \text{(B.36)}$$

Now, suppose that candidate 2 deviates to $\alpha_2 = x_2''' \in (0, \min\{r, \min(S(\alpha_1^*)\backslash\{0\})\})$, and then $\bar{U}_2(\alpha_1^*, x_2''', \bar{\gamma}_0) = p(q_2\alpha_1^*(0) + (1 - \alpha_1^*(0))) + (1 - p)$. However:

$$\bar{U}_2(\alpha_1^*, x_2''', \bar{\gamma}_0) - \bar{U}_2(\alpha_1^*, \alpha_2^*, \bar{\gamma}_0) = p\left(\frac{1}{2}(1 - q_1)\alpha_1^*(x_2') + (1 - q_2)\sum_{0<x_1<x_2'} \alpha_1^*(x_1)\right) + (1 - p)(1 - q_2) > 0. \quad \text{(B.37)}$$

That is, candidate 2 has an incentive to deviate, which is a contradiction. Likewise, we can derive a contradiction for the scenarios where $x_2' = r$ or case (ii). ■

Lemma B.4 *Consider the modified nonstrategic outlet model, and suppose that $r \leq b < |l|$. Then, there exists no equilibrium in which candidate 2 is the front-runner.*

Proof. Suppose, in contrast, that there exists an equilibrium in which candidate 2 is the front-runner. First, suppose that $0 \notin S(\alpha_1^*)$. By Lemma B.3, candidate 1's equilibrium payoff is

$\bar{U}_1(\alpha_1^*, \alpha_2^*, \bar{\gamma}_0) = (1-p)(1-q_1)$. However, if he deviates to $\alpha_1 = 0$, then:

$$\begin{aligned}\bar{U}_1(0, \alpha_2^*, \bar{\gamma}_0) &= p\left(\frac{1}{2}(1-q_1)\alpha_2^*(0) + (1-q_2)(1-\alpha_2^*(0))\right) + (1-p)(1-q_1) \\ &> \bar{U}_1(\alpha_1^*, \alpha_2^*, \bar{\gamma}_0). \end{aligned} \tag{B.38}$$

That is, candidate 2 has an incentive to deviate, which is a contradiction. Thus, $0 \in S(\alpha_1^*)$ should hold. Now, there are the following two cases to be checked.

First, consider the scenario where $q_1 > 2q_2 - 1$. Suppose, in contrast, that $0 \notin S(\alpha_2^*)$. Because of $0 \in S(\alpha_1^*)$ and Lemma B.3, candidate 2's equilibrium payoff is $\bar{U}_2(\alpha_1^*, \alpha_2^*, \bar{\gamma}_0) = p(q_2\alpha_1^*(0) + (1-\alpha_1^*(0))) + (1-p)$. Now, if candidate 2 deviates to $\alpha_2 = 0$, then:

$$\bar{U}_2(\alpha_1^*, 0, \bar{\gamma}_0) = p\left(\frac{1}{2}(1+q_1)\alpha_1^*(0) + (1-\alpha_1^*(0))\right) + (1-p). \tag{B.39}$$

However:

$$\bar{U}_2(\alpha_1^*, 0, \bar{\gamma}_0) - \bar{U}_2(\alpha_1^*, \alpha_2^*, \bar{\gamma}_0) = p\left(\frac{1}{2}(1+q_1) - q_2\right)\alpha_1^*(0) > 0, \tag{B.40}$$

where the inequality comes from $q_1 > 2q_2 - 1$ and $\alpha_1^*(0) > 0$. That is, candidate 2 has an incentive to deviate. Therefore, $0 \in S(\alpha_2^*)$ should hold. Next, suppose, in contrast, that $\alpha_2^* \neq 0$. That is, there exists $x_2' \in S(\alpha_2^*)$ such that $x_2' \neq 0$. Notice that, by Lemma B.3, $x_2' < \min\{r, x_1^-\}$ holds, where $x_1^- \in \min(S(\alpha_1^*)\backslash\{0\})$. Because $0 \in S(\alpha_2^*)$, candidate 2's equilibrium payoff is:

$$\bar{U}_2(\alpha_1^*, \alpha_2^*, \bar{\gamma}_0) = p\left(\frac{1}{2}(1+q_1)\alpha_1^*(0) + (1-\alpha_1^*(0))\right) + (1-p). \tag{B.41}$$

On the other hand, because $x_2' \in S(\alpha_2^*)$, $\bar{U}_2(\alpha_1^*, \alpha_2^*, \bar{\gamma}_0) = p(q_2\alpha_1^*(0) + (1-\alpha_1^*(0))) + (1-p)$. However:

$$\begin{aligned} &p\left(\frac{1}{2}(1+q_1)\alpha_1^*(0) + (1-\alpha_1^*(0))\right) + (1-p) - p(q_2\alpha_1^*(0) + (1-\alpha_1^*(0))) - (1-p) \\ &= p\left(\frac{1}{2}(1+q_1) - q_2\right)\alpha_1^*(0) > 0, \end{aligned} \tag{B.42}$$

where the inequality comes from $q_1 > 2q_2 - 1$ and $\alpha_1^*(0) > 0$, which is a contradiction. Therefore, $\alpha_2^* = 0$ should hold. Finally, suppose, in contrast, that $\alpha_1^* \neq 0$. That is, there exists $x_1' \in S(\alpha_1^*)$ such that $x_1' \neq 0$. Because $0 \in S(\alpha_1^*)$, candidate 1's equilibrium payoff is $\bar{U}_1(\alpha_1^*, \alpha_2^*, \bar{\gamma}_0) = p(1-q_1)/2 + (1-p)(1-q_1)$. On the other hand, because $x_1' \in S(\alpha_1^*)$, $\bar{U}_1(\alpha_1^*, \alpha_2^*, \bar{\gamma}_0) = (1-p)(1-q_1)$ also should hold. However, the payoff from $x_1 = 0$ is strictly greater than that from $x_1 = x_1'$, which is a contradiction. Thus, $\alpha_1^* = 0$ must hold. However, by Assumption B.4-(i), given $\alpha_1^* = \alpha_2^* = 0$, the voter's best response to the suppressed message is $\gamma^*(\phi) = (1, 0)$, which is a contradiction to that candidate 2 is the front-runner.

Second, consider the scenario where $q_1 < 2q_2 - 1$. Suppose, in contrast, that $0 \notin S(\alpha_2^*)$. Because $0 \in S(\alpha_1^*)$, candidate 1's equilibrium payoff is $\bar{U}_1(\alpha_1^*, \alpha_2^*, \bar{\gamma}_0) = p(1 - q_2) + (1 - p)(1 - q_1)$. Now, if candidate 1 deviates to $\alpha_1 = x_2^- \in \min S(\alpha_2^*)$, then:

$$\bar{U}_1(x_2^-, \alpha_2^*, \bar{\gamma}_0) = p\left(\frac{1}{2}(1 - q_1)\alpha_2^*(x_2^-) + (1 - q_2)(1 - \alpha_2^*(x_2^-))\right) + (1 - p)(1 - q_1). \tag{B.43}$$

However:

$$\bar{U}_2(x_2^-, \alpha_2^*, \bar{\gamma}_0) - \bar{U}_2(\alpha_1^*, \alpha_2^*, \bar{\gamma}_0) = p\left(\frac{1}{2}(1 - q_1) - (1 - q_2)\right)\alpha_2^*(x_2^-) > 0, \tag{B.44}$$

where the inequality comes from $q_1 < 2q_2 - 1$ and $\alpha_2^*(x_2^-) > 0$. That is, candidate 1 has an incentive to deviate, which is a contradiction. Thus, $0 \in S(\alpha_2^*)$ should hold. Because $0 \in S(\alpha_2^*)$, candidate 2's equilibrium payoff is:

$$\bar{U}_2(\alpha_1^*, \alpha_2^*, \bar{\gamma}_0) = p\left(\frac{1}{2}(1 + q_1)\alpha_1^*(0) + (1 - \alpha_1^*(0))\right) + (1 - p). \tag{B.45}$$

Now, if candidate 2 deviates to $\alpha_2 = x_2' \in (0, \min\{r, x_1^-\})$, then $\bar{U}_2(\alpha_1^*, x_2', \bar{\gamma}_0) = p(q_2\alpha_1^*(0) + (1 - \alpha_1^*(0))) + (1 - p)$. However:

$$\bar{U}_2(\alpha_1^*, x_2', \bar{\gamma}_0) - \bar{U}_2(\alpha_1^*, \alpha_2^*, \bar{\gamma}_0) = p\left(q_2 - \frac{1}{2}(1 + q_1)\right)\alpha_1^*(0) > 0, \tag{B.46}$$

where the inequality comes from $q_1 < 2q_2 - 1$ and $\alpha_1^*(0) > 0$. That is, candidate 2 has an incentive to deviate, which is a contradiction. ■

Lemma B.5 *Consider the modified nonstrategic outlet model, and suppose that $r \leq b < |l|$. Then, there exists no equilibrium in which there exists no front-runner.*

Proof. Suppose, in contrast, that there exists an equilibrium without the front-runner. First, suppose, in contrast, that $\alpha_1^* \neq 0$. That is, there exists $x_1' \in S(\alpha_1^*)$ such that $x_1' \neq 0$. Because

$x_1' \in S(\alpha_1^*)$, candidate 1's equilibrium payoff is:

$$\bar{U}_1(\alpha_1, \alpha_2^*, \bar{\gamma}_{1/2}) = p\left(\frac{1}{2}\alpha_2^*(x_1') + \left(1 - \frac{1}{2}q_2\right)\sum_{x_2 > x_1'} \alpha_2^*(x_2) + \frac{1}{2}q_2 \sum_{x_2 < x_1'} \alpha_2^*(x_2)\right) + (1-p)\left(1 - \frac{1}{2}q_1\right). \tag{B.47}$$

Now, if candidate 1 deviates to $\alpha_1 = 0$, then:

$$\bar{U}_1(0, \alpha_2^*, \bar{\gamma}_{1/2}) = p\left(\frac{1}{2}\alpha_2^*(0) + \left(1 - \frac{1}{2}q_2\right)(1 - \alpha_2^*(0))\right) + (1-p)\left(1 - \frac{1}{2}q_1\right). \tag{B.48}$$

Notice that:

$$\bar{U}_1(0, \alpha_2^*, \bar{\gamma}_{1/2}) - \bar{U}_1(\alpha_1^*, \alpha_2^*, \bar{\gamma}_{1/2}) = p\left(\frac{1}{2}(1-q_2)\alpha_2^*(0) + \frac{1}{2}(1-q_2)\alpha_2^*(x_1') + (1-q_2)\sum_{0 < x_2 < x_1'} \alpha_2^*(x_2)\right). \tag{B.49}$$

Thus, to hold this equilibrium, $S(\alpha_2^*) \subset (x_1^+, b]$ must hold, where $x_1^+ \in \max S(\alpha_1^*)$; otherwise, candidate 1 deviates to $\alpha_1 = 0$. Then, because $x_2 \in S(\alpha_2^*)$, candidate 2's equilibrium payoff is:

$$\bar{U}_2(\alpha_1^*, \alpha_2^*, \bar{\gamma}_{1/2}) = \frac{1}{2}pq_2 + (1-p)\left(\left(1 - \frac{1}{2}q_2\right)\mathbb{1}(x_2 < r) + \frac{1}{2}q_2\mathbb{1}(x_2 > r) + \frac{1}{2}\mathbb{1}(x_2 = r)\right), \tag{B.50}$$

where $\mathbb{1}(\cdot)$ represents a indicator function. If candidate 2 deviates to $\alpha_2 = 0$, then:

$$\bar{U}_2(\alpha_1^*, 0, \bar{\gamma}_{1/2}) = p\left(\frac{1}{2}\alpha_1^*(0) + \left(1 - \frac{1}{2}q_2\right)(1 - \alpha_1^*(0))\right) + (1-p)\left(1 - \frac{1}{2}q_2\right). \tag{B.51}$$

However:

$$\bar{U}_2(\alpha_1^*, 0, \bar{\gamma}_{1/2}) - \bar{U}_2(\alpha_1^*, \alpha_2^*, \bar{\gamma}) \geq p\left(\frac{1}{2}(1-q_2)\alpha_1^*(0) + (1-q_2)(1 - \alpha_1^*(0))\right) > 0, \tag{B.52}$$

where the first inequality comes from $1 - q_2 > 1/2 > q_2/2$. That is, candidate 2 has an incentive to deviate, which is a contradiction. Thus, $\alpha_1^* = 0$ should hold.

Next, suppose, in contrast, that $\alpha_2^* \neq 0$. That is, there exists $x_2' \in S(\alpha_2^*)$ such that $x_2' \neq 0$. Because $x_2' \in S(\alpha_2^*)$ and $\alpha_1^* = 0$, candidate 2's equilibrium payoff is given by (B.50). Now, if candidate 2 deviates to $\alpha_2 = 0$, then:

$$\bar{U}_2(\alpha_1^*, 0, \bar{\gamma}_{1/2}) = \frac{1}{2}p + (1-p)\left(1 - \frac{1}{2}q_2\right). \tag{B.53}$$

However:

$$\bar{U}_2(\alpha_1^*, 0, \bar{\gamma}_{1/2}) - \bar{U}_2(\alpha_1^*, \alpha_2^*, \bar{\gamma}_{1/2}) \geq p\left(\frac{1}{2}(1-q_2)\right) > 0, \tag{B.54}$$

where the first inequality comes from $1-q_2/2 > 1/2 > q_2/2$. That is, candidate 2 has an incentive to deviate, which is a contradiction. Therefore, $\alpha_2 = 0$ should hold. However, because of Assumption B.4-(i), given $\alpha_1^* = \alpha_2^* = 0$, the voter's best response to the suppressed message is $\gamma^*(\phi) = (1, 0)$, which is a contradiction. ■

These lemmas insist that we can focus on the equilibria in which candidate 1 is the front-runner. We then characterize the set of equilibria as follows.

Proposition B.10 *Consider the modified nonstrategic outlet model, and assume that $r \leq b < |l|$.*

(i) If $q_1 > 2q_2 - 1$, then there exists a $(0,0)$ equilibrium, and it is the unique equilibrium. Then, the degree of distortion is $d(q_1, q_2) = p(1-p)q_2 r$.

(ii) If $\bar{q}_1 < q_1 < 2q_2 - 1$, then there exists no equilibrium.

(iii) Suppose that $q_1 < \bar{q}_1$.

(a) If there exists an equilibrium, then $S(\alpha_1^*) \subset [0, r)$ and $\alpha_2^* = r$.

(b) Suppose that $p \geq 1/2$. Then, there exists an equilibrium e with $d(e; q_1, q_2) = d$ if and only if $\underline{D}(q_1, q_2) < d < \overline{D}(q_1, q_2)$, where $\underline{D}(q_1, q_2) = p(1-p)r$ and $\overline{D}(q_1, q_2) = p(2-p)r$.

Proof. (i) (Existence) We show that there exists the desired equilibrium in which the induced outcome of the news-reporting stage is $\bar{\gamma}_1$. First, we show the optimality of $\gamma^*(\phi) = (1, 0)$. Given $\alpha_1^* = \alpha_2^* = 0$, the voter's expected utility from $y = y_1$ and y_2 are $-(1-p)(pq_2 + (1-p)q_1)r$ and $-(1-p)q_1|l|$, respectively. By Assumption B.4-(i), $\gamma^*(\phi) = (1, 0)$ is the best response. Next, we show the optimality of $\alpha_1^* = 0$. Given α_2^* and $\bar{\gamma}_1$, candidate 1's equilibrium payoff is $\bar{U}_1(\alpha_1^*, \alpha_2^*, \bar{\gamma}_1) = p(1+q_1)/2 + (1-p)$. Now, if candidate 1 deviates to $\alpha_1 = x_1' \neq 0$, then his winning probability is $\bar{U}_1(x_1', \alpha_2^*, \bar{\gamma}_1) = pq_2 + (1-p)$. Because $q_1 > 2q_2 - 1$, $\alpha_1^* = 0$ is the best response. Finally, we show the optimality of α_2^*. Given α_1^* and $\bar{\gamma}_1$, candidate 2's equilibrium payoff is $\bar{U}_2(\alpha_1^*, \alpha_2^*, \bar{\gamma}_1) = p(1-q_1)/2 + (1-p)(1-q_2)$. If candidate 2 deviates to either $\alpha_2 = r$, $x_2' \in (0, r)$, or $x_2'' \in (r, b]$, then his winning probabilities are as follows:

- $\bar{U}_2(\alpha_1^*, r, \bar{\gamma}_1) = (1-p)(1-q_1)/2$;

- $\bar{U}_2(\alpha_1^*, x_1', \bar{\gamma}_1) = (1-p)(1-q_2)$;

- $\bar{U}_1(\alpha_1^*, x_1'', \bar{\gamma}_1) = 0$.

Because $q_1 > 2q_2 - 1$, $\alpha_2^* = 0$ is the best response. Therefore, it is a PBE.

(Uniqueness) Suppose, in contrast, that there exists an equilibrium in which either $\alpha_1^* \neq 0$ or $\alpha_2^* \neq 0$. Without loss of generality, we assume that $\alpha_1^* \neq 0$. That is, there exists $x_1' \in S(\alpha_1^*)$ such that $x_1' \neq 0$. First, suppose, in contrast, that $0 \notin S(\alpha_2^*)$. By Lemmas B.4 and B.5, the front-runner should be candidate 1 in this equilibrium. Hence, by Lemma B.2, candidate 2 never wins when candidate 1 is the opportunistic type. Then, we say that $\bar{U}_2(\alpha_1^*, \alpha_2^*, \bar{\gamma}_1) \leq (1-p)(1-q_2)$, where the inequality comes from the fact that $q_1 > 2q_2 - 1$. However, if candidate 2 deviates to $\alpha_2 = 0$, then:

$$\begin{aligned}\bar{U}_2(\alpha_1^*, 0, \bar{\gamma}_1) &= p\left(\frac{1}{2}(1-q_1)\alpha_1^*(0) + (1-q_2)(1-\alpha_1^*(0))\right) + (1-p)(1-q_2) \\ &> (1-p)(1-q_2) \geq \bar{U}_2(\alpha_1^*, \alpha_2^*, \bar{\gamma}_1). \end{aligned} \tag{B.55}$$

That is, candidate 2 has an incentive to deviate, which is a contradiction. Hence, $0 \in S(\alpha_2^*)$ should hold. Because of $x_1' \in S(\alpha_1^*)$ and Lemma B.2, candidate 1's equilibrium payoff is $\bar{U}_1(\alpha_1^*, \alpha_2^*, \bar{\gamma}_1) = p(q_2\alpha_2^*(0) + (1-\alpha_2^*(0))) + (1-p)$. However, if candidate 1 deviates to $\alpha_1 = 0$, then:

$$\bar{U}_1(0, \alpha_2^*, \bar{\gamma}_1) = p\left(\frac{1}{2}(1+q_1)\alpha_2^*(0) + (1-\alpha_2^*(0))\right) + (1-p) > \bar{U}_1(\alpha_1^*, \alpha_2^*, \bar{\gamma}_1), \tag{B.56}$$

where the inequality comes from $q_1 > 2q_2 - 1$. That is, candidate 1 has an incentive to deviate, which is a contradiction. Therefore, we conclude that the $(0,0)$ equilibrium is the unique equilibrium. Therefore, it is straightforward that the degree of distortion of the $(0,0)$ equilibrium is $d(q_1, q_2) = p(1-p)q_2 r$.

(ii) Suppose, in contrast, that there exists an equilibrium. By Lemmas B.4 and B.5, candidate 1 should be the front-runner in this equilibrium. First, suppose, in contrast, that $0 \notin S(\alpha_2^*)$. Hence, by Lemma B.2, candidate 2's equilibrium payoff is $\bar{U}_2(\alpha_1^*, \alpha_2^*, \bar{\gamma}_1) \leq (1-p)(1-q_1)/2$ because of $q_1 < 2q_2 - 1$. However, if candidate 2 deviates to $\alpha_1 = 0$, then:

$$\begin{aligned}\bar{U}_2(\alpha_1^*, 0, \bar{\gamma}_1) &= p\left(\frac{1}{2}(1-q_1)\alpha_1^*(0) + (1-q_2)(1-\alpha_1^*(0))\right) + (1-p)(1-q_2) \\ &= p\left(\frac{1}{2}(1-q_1) - (1-q_2)\right)\alpha_1^*(0) + (1-q_2) \\ &\geq 1 - q_2 > \frac{1}{2}(1-p)(1-q_1) \geq \bar{U}_2(\alpha_1^*, \alpha_2^*, \bar{\gamma}_1), \end{aligned} \tag{B.57}$$

where the first and second inequalities come from $q_1 < 2q_2 - 1$ and $q_1 > \bar{q}_1$, respectively. That is, candidate 2 has an incentive to deviate, which is a contradiction. Therefore, $0 \in S(\alpha_2^*)$ should hold.

Second, suppose, in contrast, that $0 \in S(\alpha_1^*)$. Hence, candidate 1's equilibrium payoff is $\bar{U}_1(\alpha_1^*, \alpha_2^*, \bar{\gamma}_1) = p((1+q_1)\alpha_2^*(0)/2 + (1-\alpha_2^*(0))) + (1-p)$. Now, if candidate 1 deviates to $\alpha_1 = x_1' < \min(S(\alpha_2^*)\backslash\{0\})$, then $\bar{U}_1(x_1', \alpha_2^*, \bar{\gamma}_1) = p(q_2\alpha_2^*(0) + (1-\alpha_2^*(0))) + (1-p)$. However:

$$\bar{U}_1(x_1', \alpha_2^*, \bar{\gamma}_1) - \bar{U}_1(\alpha_1^*, \alpha_2^*, \bar{\gamma}_1) = p\left(q_2 - \frac{1}{2}(1+q_1)\right)\alpha_2^*(0) > 0, \tag{B.58}$$

where the inequality comes from $q_1 < 2q_2 - 1$ and $\alpha_2^*(0) > 0$. That is, candidate 1 has an incentive to deviate, which is a contradiction. Therefore, $0 \notin S(\alpha_1^*)$ should hold.

Third, suppose, in contrast, that $\alpha_2^* \neq 0$; that is, there exists $x_2'' \in S(\alpha_2^*)$ such that $x_2'' \neq 0$. Notice that, by Lemma B.2, $x_2'' \notin S(\alpha_1^*)$, and candidate 2 never wins when he proposes $x_2 = x_2''$ and candidate 1 is the opportunistic type. Because $0 \notin S(\alpha_1^*)$ and $0 \in S(\alpha_2^*)$, candidate 2's equilibrium payoff is $\bar{U}_2(\alpha_1^*, \alpha_2^*, \bar{\gamma}_1) = 1 - q_2$. There are the following cases to be checked.

Case (i): $x_2'' \in (0, r)$.

However, $\bar{U}_2(\alpha_1^*, x_2'', \bar{\gamma}_0) = (1-p)(1-q_2) < \bar{U}_2(\alpha_1^*, \alpha_2^*, \bar{\gamma}_1)$. Thus, $x_2'' \notin (0, r)$.

Case (ii): $x_2'' = r$.

However, $\bar{U}_2(\alpha_1^*, x_2'', \bar{\gamma}_1) = (1-p)(1-q_1)/2 < \bar{U}_2(\alpha_1^*, \alpha_2^*, \bar{\gamma}_1)$, where the inequality comes from $q_1 > \bar{q}_1$. Thus, $x_2'' \neq r$.

Case (iii): $x_2'' \in (r, b]$.

However, $\bar{U}_2(\alpha_1^*, x_2'', \bar{\gamma}_1) = 0 < \bar{U}_2(\alpha_1^*, \alpha_2^*, \bar{\gamma}_1)$. Thus, $x_2'' \notin (r, b]$.

Therefore, $\alpha_2^* = 0$, should hold.

Forth, suppose, in contrast, that $\bar{x}_1^- \in \min S(\alpha_1^*) \leq r$. Notice that because $0 \notin S(\alpha_1^*)$, $\bar{x}_1^- > 0$ and $\bar{U}_2(\alpha_1^*, \alpha_2^*, \bar{\gamma}_1) = 1 - q_2$ hold. There are the following two cases to be checked.

Case (i): $\bar{x}_1^- < r$.

However, if candidate 2 deviates to $\alpha_2 = \bar{x}_1^-$, then:

$$\begin{aligned} \bar{U}_2(\alpha_1^*, \bar{x}_1^-, \bar{\gamma}_1) &= p\left(\frac{1}{2}(1-q_1)\alpha_1^*(\bar{x}_1^-) + (1-q_2)(1-\alpha_1^*(\bar{x}_1^-))\right) + (1-p)(1-q_2) \\ &= p\left(\frac{1}{2}(1-q_1) - (1-q_2)\right)\alpha_1^*(\bar{x}_1^-) + (1-q_2) > \bar{U}_2(\alpha_1^*, \alpha_2^*, \bar{\gamma}_1), \end{aligned} \tag{B.59}$$

where the inequality comes from $q_1 < 2q_2 - 1$ and $\alpha_1^*(\bar{x}_1^-) > 0$.

Case (ii): $\bar{x}_1^- = r$.

However, if candidate 2 deviates to $\alpha_2 = r$, then:

$$\begin{aligned}\bar{U}_2(\alpha_1^*, r, \bar{\gamma}_1) &= p\left(\frac{1}{2}(1-q_1)\alpha_1^*(r) + (1-q_2)(1-\alpha_1^*(r))\right) + \frac{1}{2}(1-p)(1-q_1) \\ &= p\left(\frac{1}{2}(1-q_1) - (1-q_2)\right)\alpha_1^*(r) + p(1-q_2) + \frac{1}{2}(1-p)(1-q_1) \\ &> \bar{U}_2(\alpha_1^*, \alpha_2^*, \bar{\gamma}_1),\end{aligned} \tag{B.60}$$

where the inequality comes from $q_1 < 2q_2 - 1$ and $\alpha_1^*(r) > 0$.

That is, in each case, candidate 2 has an incentive to deviate, which is a contradiction. Thus, $S(\alpha_1^*) \subset (r, b]$ should hold.

Now, if candidate 2 deviates to $\alpha_2 = r$, then $\bar{U}_2(\alpha_1^*, r, \bar{\gamma}_1) = p(1-q_2) + (1-p)(1-q_1)/2$. However:

$$\bar{U}_2(\alpha_1^*, r, \bar{\gamma}_1) - \bar{U}_2(\alpha_1^*, \alpha_2^*, \bar{\gamma}_1) = (1-p)\left(\frac{1}{2}(1-q_1) - (1-q_2)\right) > 0, \tag{B.61}$$

where the inequality comes from $q_1 < 2q_2 - 1$. That is, candidate 2 has an incentive to deviate, which is a contradiction. Therefore, there exists no equilibrium.

(iii)-(a). Suppose, in contrast, that there exists an equilibrium in which either (i) $S(\alpha_1^*) \cap [r, b] \neq \emptyset$ or (ii) $\alpha_2^* \neq r$. First, we suppose that $\alpha_2^* \neq r$; that is, there exists $x_2' \in S(\alpha_2^*)$ such that $x_2' \neq r$. By Lemmas B.4 and B.5, candidate 1 should be the front-runner. Now, suppose, in contrast, that $0 \notin S(\alpha_2^*)$. Then, by Lemma B.2, $x_1 < \min S(\alpha_2^*)$ holds for any $x_1 \in S(\alpha_1^*)$. Therefore, candidate 2 never wins the election in the equilibrium when candidate 1 is the opportunistic type. Because $x_2' \in S(\alpha_2^*)$ and $x_2' \neq r$, candidate 2's equilibrium payoff is $\bar{U}_2(\alpha_1^*, \alpha_2^*, \bar{\gamma}_1) \leq (1-p)(1-q_1)$. However, if candidate 2 deviates to $\alpha_2 = r$, then:

$$\begin{aligned}\bar{U}_2(\alpha_1^*, r, \bar{\gamma}_1) &= p\left(\frac{1}{2}(1-q_1)\alpha_1^*(r) + (1-q_2)\sum_{x_1 > r}\alpha_1^*(x_1)\right) + \frac{1}{2}(1-p)(1-q_1) \\ &> (1-p)(1-q_2) \geq \bar{U}_2(\alpha_1^*, \alpha_2^*, \bar{\gamma}_1),\end{aligned} \tag{B.62}$$

where the first inequality comes from $q_1 < 2q_2 - 1$. That is, candidate 2 has an incentive to deviate. which is a contradiction. Therefore, $0 \in S(\alpha_2^*)$ should hold.

Next, suppose, in contrast, that $0 \in S(\alpha_1^*)$. Because $0 \in S(\alpha_1^*) \cap S(\alpha_2^*)$, candidate 1's equilibrium payoff is $\bar{U}_1(\alpha_1^*, \alpha_2^*, \bar{\gamma}_1) = p((1+q_1)\alpha_2^*(0)/2 + (1-\alpha_2^*(0))) + (1-p)$. If candidate 1 deviates to

$\alpha_1 = x_1' < \min(S(\alpha_2^*)\backslash\{0\})$, then $\bar{U}_1(x_1', \alpha_2^*, \bar{\gamma}_1) = p(q_2\alpha_2^*(0) + (1-\alpha_2^*(0))) + (1-p)$. However:

$$\bar{U}_1(x_1', \alpha_2^*, \bar{\gamma}_1) - \bar{U}_1(\alpha_1^*, \alpha_2^*, \bar{\gamma}_1) = p\left(q_2 - \frac{1}{2}(1+q_1)\right)\alpha_2^*(0) > 0, \tag{B.63}$$

where the inequality comes from $q_1 < 2q_2 - 1$ and $\alpha_2^*(0) > 0$. That is, candidate 1 has an incentive to deviate, which is a contradiction. Thus, $0 \notin S(\alpha_1^*)$ should hold.

Because $0 \notin S(\alpha_1^*)$ and $0 \in S(\alpha_2^*)$, candidate 2's equilibrium payoff is $\bar{U}_2(\alpha_1^*, \alpha_2^*, \bar{\gamma}_1) = 1 - q_2$. However, if candidate 2 deviates to $\alpha_2 = r$, then:

$$\begin{aligned}\bar{U}_2(\alpha_1^*, r, \bar{\gamma}_1) &= p\left(\frac{1}{2}(1-q_1)\alpha_1^*(r) + (1-q_2)\sum_{x_1>r}\alpha_1^*(x_1)\right) + \frac{1}{2}(1-p)(1-q_1) \\ &\geq \frac{1}{2}(1-p)(1-q_1) > \bar{U}_2(\alpha_1^*, \alpha_2^*, \bar{\gamma}_1),\end{aligned} \tag{B.64}$$

where the second inequality comes from $q_1 < \bar{q}_1$. That is, candidate 2 has an incentive to deviate, which is a contradiction. Therefore, $\alpha_2^* = r$ must hold, which implies that $S(\alpha_1^*) \cap [r, b] \neq \emptyset$ should hold. Because there exists $x_1'' \in S(\alpha_1^*) \cap [r, b]$, candidate 1's equilibrium payoff is $\bar{U}_1(\alpha_1^*, \alpha_2^*, \bar{\gamma}_1) = pq_2 + (1-p) < 1$. However, if candidate 1 deviates to $\alpha_1 = 0$, then $\bar{U}_1(0, \alpha_2^*, \bar{\gamma}_1) = 1$. That is, candidate 1 has an incentive to deviate, which is a contradiction. Therefore, if there exists an equilibrium, then $S(\alpha_1^*) \subset [0, r)$ and $\alpha_2^* = r$ should hold.

(iii)-(b). (Necessity) Fix an equilibrium $e = (\alpha_1^*, \alpha_2^*, \gamma^*)$, arbitrarily. Let $\bar{x}_1 \equiv \sum_{x_1 \in X} x_1\alpha_1^*(x_1)$ be the expected policy of candidate 1 conditional being on the opportunistic type. Because of Lemmas B.4 and B.5, candidate 1 should be the front-runner in this equilibrium. Furthermore, because of Proposition B.10-(iii)-1, $S(\alpha_1^*) \subset [0, r)$ and $\alpha_2^* = r$ must hold. First, suppose, in contrast, that $\alpha_1^* = 0$. Hence, candidate 2's equilibrium payoff is $\bar{U}_2(\alpha_1^*, \alpha_2^*, \bar{\gamma}_1) = (1-p)(1-q_1)/2$. Now, if candidate 2 deviates to $\alpha_2 = 0$, then $\bar{U}_2(\alpha_1^*, 0, \bar{\gamma}_1) = p(1-q_1)/2 + (1-p)(1-q_2)$. However:

$$\bar{U}_2(\alpha_1^*, 0, \bar{\gamma}_1) - \bar{U}_2(\alpha_1^*, \alpha_2^*, \bar{\gamma}_1) = \frac{1}{2}(1-q_1)(2p-1) + (1-p)(1-q_2) > 0, \tag{B.65}$$

where the inequality comes from $p \geq 1/2$. That is, candidate 2 has an incentive to deviate. Thus, $\alpha_1^* \neq 0$. Because $S(\alpha_1^*) \subset [0, r)$ and $\alpha_1^* \neq 0$, $\bar{x}_1 \in (0, r)$. Hence, the degree of distortion of equilibrium e is:

$$d(e; q_1, q_2) = p^2\bar{x}_1 + p(1-p)r + p(1-p)\bar{x}_1 = p\bar{x}_1 + p(1-p)r. \tag{B.66}$$

Therefore, because $\bar{x}_1 \in (0, r)$, $\underline{D}(q_1, q_2) < d(e; q_1, q_2) < \overline{D}(q_1, q_2)$ holds.

(Sufficiency) Fix $d \in (\underline{D}(q_1, q_2), \overline{D}(q_1, q_2))$, arbitrarily. Now, we show that the following is a PBE: (i) α_1^* satisfies the following conditions:

- $S(\alpha_1^*) = \{x - \delta, x + \delta, x - 2\delta, x + 2\delta, \cdots, x - (N/2)\delta, x + (N/2)\delta\}$, where $x \equiv d/p - (1-p)r$, N is an even number such that:

$$N \geq \frac{p((1-q_1) - 2(1-q_2))}{(1-p)(1-q_1) - 2(1-q_2)}, \tag{B.67}$$

  and $\delta > 0$ is so small that $(N/2)\delta$ is sufficiently close to 0;

- $\alpha_1^*(x_1) = 1/N$ for any $x_1 \in S(\alpha_1^*)$;

(ii) $\alpha_2^* = r$; and (iii) $\gamma^*(\phi) = (1, 0)$.

First, we show that optimality of γ^*. By construction, notice that $\bar{x}_1 = x$. Hence, given α_1^*, α_2^*, and $m = \phi$, the voter's expected payoffs from action $y = y_1$ and y_2 are $-p(pq_2+(1-p)q_1)x-(1-p)q_1r$ and $-p(pq_2 + (1-p)q_1)r - (1-p)q_1|l|$, respectively. Because of $x < r < |l|$:

$$\begin{aligned} -p(pq_2 + (1-p)q_1)x - (1-p)q_1r &> -p(pq_2 + (1-p)q_1)r - (1-p)q_1r \\ &> -p(pq_2 + (1-p)q_1)r - (1-p)q_1|l|. \end{aligned} \tag{B.68}$$

That is, $\gamma^*(\phi) = (1, 0)$ is the best response.

Next, we show the optimality of α_1^*. Notice that by construction, $x_1 \in (0, r)$ for any $x_1 \in S(\alpha_1^*)$. Given α_2^* and $\bar{\gamma}_1$, if candidate 1 proposes policy $x_1 \in (0, 1)$, then $\bar{U}_1(x_1, \alpha_2^*, \bar{\gamma}_1) = 1$. Therefore, it is obvious that candidate 1 has no incentive to deviate from α_1^*.

Finally, we show the optimality of α_2^*. Given α_1^* and $\bar{\gamma}_1$, candidate 2's equilibrium payoff is $\bar{U}_2(\alpha_1^*, \alpha_2^*, \bar{\gamma}_1) = (1-p)(1-q_1)/2$. Now, suppose that candidate 2 deviates to $\alpha_2 = x_2'$. There are the following cases to be checked.

Case (i): $x_2' = 0$.

In this scenario, $\bar{U}_2^*(\alpha_1^*, 0, \bar{\gamma}_1) = 1 - q_2$. However, because $q_1 < \bar{q}_1$, $\bar{U}_2(\alpha_1^*, \alpha_2^*, \bar{\gamma}_1) > \bar{U}_2(\alpha_1^*, 0, \bar{\gamma}_1)$ holds.

Case (ii): $x_2' \in (0, r) \backslash S(\alpha_1^*)$.

In this scenario, $\bar{U}_2(\alpha_1^*, x_2', \bar{\gamma}_1) = p(1-q_2)\sum_{x_1 > x_2'} \alpha_1^*(x_1) + (1-p)(1-q_2) < 1 - q_2$. Again, because of $q_1 < \bar{q}_1$, $\bar{U}_2(\alpha_1^*, \alpha_2^*, \bar{\gamma}_1) > \bar{U}_2(\alpha_1^*, 0, \bar{\gamma}_1)$ holds.

Case (iii): $x_2' \in (0, r) \cap S(\alpha_1^*)$.

By construction:

$$\begin{aligned}\bar{U}_2(\alpha_1^*, x_2', \bar{\gamma}_1) &= p\left(\frac{1}{2}(1-q_1)\alpha_1^*(x_2') + (1-q_2)\sum_{x_1 > x_2'} \alpha_1^*(x_1)\right) + (1-p)(1-q_1) \\ &\leq \frac{p}{N}\left(\frac{1}{2}(1-q_1) - (1-q_2)\right) + (1-q_2) \\ &\leq \frac{1}{2}(1-p)(1-q_1) = \bar{U}_2(\alpha_1^*, \alpha_2^*, \bar{\gamma}_1),\end{aligned} \tag{B.69}$$

where the first inequality comes from $q_1 < 2q_2 - 1$ with the equality when $x_2' = x - \delta$, and the second inequality comes from (B.67).

Case (iv): $x_2' \in (r, b]$.

In this scenario, $\bar{U}_2(\alpha_1^*, 0, \bar{\gamma}_1) = 0 < \bar{U}_2(\alpha_1^*, \alpha_2^*, \bar{\gamma}_1)$ holds.

Therefore, candidate 2 has no incentive to deviate. Thus, it is a PBE. Furthermore, by construction, the degree of distortion of this equilibrium is:

$$d(e; q_1, q_2) = p\left(\frac{d}{p} - (1-p)r\right) + p(1-p)r = d. \tag{B.70}$$

Thus, we conclude that there exists an equilibrium whose degree of distortion is $d \in (\underline{D}(q_1, q_2), \overline{D}(q_1, q_2))$.
■

As in the model in which $q_1 = q_2$, the uniqueness of the $(0, 0)$ equilibrium appears when $q_1 > 2q_2 - 1$, whose underlying mechanism is similar to those mentioned in the body of the paper. That is, because q_1 and q_2 are sfficiently close, the winning probability conditional on suppression do not drastically change depending on whether the policy pair is in the agreement or disagreement regions. Hence, each candidate prioritizes the event at which the information is disclosed, and then he has a strong incentive to appeal to the voter to win the election when the information is disclosed. As a result, only the $(0, 0)$ equilibrium exists.

On the contrary, if $q_1 < \bar{q}_1$, then there exist multiple equilibria. Notice that this scenario is closer to the baseline model in the sense that the manipulation probability is sufficiently divergent depending on whether the policy pair is in the agreement regions or not. Hence, in contrast to the previous scenario, the candidates have stronger self-mediatization incentives, as in the baseline model. In other words, the front-runner has an incentive to propose a policy that induces policy divergence to enjoy his advantage under suppression. By contrast, the underdog has an incentive to propose a policy that induces policy convergence to mitigate the disadvantage under suppression.

An easy way to resolve such a zero-sum-game structure is constructing a no-front-runner equilibrium; that is, the winning probability should be identical for any policy pair $z \in [0, b]^2$ independent of whether the policies are convergent or not. However, as shown in Lemma B.5, such an equilibrium never exists. Therefore, an equilibrium should have the following structure: candidate 2, the underdog, proposes policy $x_2 = r$ for certain to induce the tie when candidate 1 is the ideological type, whereas candidate 1, the front-runner, randomizes policies in $(0, r)$ to avoid the policy convergence and the loss under disclosure. In fact, we can construct multiple equilibria satisfying this property, as demonstrated in Proposition B.10-(iii)-(b).

It is worthwhile to remark that the distortion is also underestimated when $q_1 < \bar{q}_1$. As shown in Proposition B.10-(iii)-(b), the lower bound is identical to that in the baseline model. In contrast to the scenario in which $q_1 > 2q_2 - 1$, there is no direct distortion. That is, because the voter's decision is correct for any policy pair on the equilibrium path, whether the information is stochastically disclosed or not is irrelevant to the outcome. Furthermore, because $\alpha_2^* = r$ must hold in any equilibrium, the equilibrium distortion conditional on $\theta_1 = I$ and $\theta_2 = O$ should be r. Therefore, by taking α_1^* whose expectation is sufficiently close to 0, the equilibrium distortion converges to that of the infimum of the baseline model. In other words, the misspecification is negligible for the lower bound. However, it is nonnegligible for the upper bound. Notice that there exists an equilibrium in which both players propose policies that are greater than r in the baseline model because the no-front-runner structure can be sustainable in equilibrium. On the contrary, however, this structure is impossible in this reduced form scenario, and then the equilibrium policies should be bounded above by r. As a result, the upper bound is underestimated. Therefore, we conclude that the nonstrategic outlet model still generates nonnegligible misspecification even though it is close to the baseline model.

Furthermore, we want to emphasize that such a reduced form model has a potentially technical problem. That is, there exists no equilibrium when $\bar{q}_1 < q_1 < 2q_2 - 1$, as demonstrated in Proposition B.10-(ii) because of the following two reasons. First, because there exists a nonnegligible difference between q_1 and q_2, each candidate has a strong self-mediatization incentive. Therefore, as in the case of $q_1 < \bar{q}_1$, the zero-sum-game structure appears among the candidates. Second, however, because the difference between q_1 and q_2 is intermediate, candidate 2, the underdog, cannot ignore the loss under disclosure. In other words, candidate 2 prefers the victory under disclosure by proposing a smaller policy than the opponent to the draw against the ideological candidate 1 by proposing policy $x_2 = r$. That is, the candidates also have a strong incentive to appeal to the voter. The coexistence of these two strong incentives is the origin of the problem. In contrast to the previous case, the zero-sum-game structure cannot be avoided by $\alpha_2 = r$. Therefore, to construct an equilibrium, it is necessary that the equilibrium has no front-runner to resolve the zero-sum-game structure. However, this is impossible, as shown in Lemma B.5. As a result, we cannot find an equilibrium in this scenario.