Multidimensional Cheap Talk with Sequential Messages *

Shintaro Miura[†]

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Abstract

This paper studies a two-dimensional cheap talk game with two senders and one receiver. The senders possess the same information and sequentially send messages about that information. In one-dimensional sequential message cheap talk games where the state space is unbounded, the information is fully transmitted under the self-serving belief, as suggested by Krishna and Morgan (2001b). However, this result depends crucially on the structure of the one-dimensional model. It generally does not hold in two-dimensional models. We consider the extended self-serving belief, which implies full information transmission even if the self-serving belief cannot work. Then, we show that the necessary and sufficient condition for the existence of the fully revealing equilibrium is that the senders have opposing-biased preferences.

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Key Words: multidimensional cheap talk; multi-senders; sequential communication; fully revealing equilibrium.

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[†]Department of Economics, Kanagawa University, 3-27-1, Rokkakubashi, Kanagawa-ku, Yokohama, Kanagawa, 221-8686 JAPAN. E-mail: smiura@kanagawa-u.ac.jp

1 Introduction

This paper studies a cheap talk game between two senders and one receiver with a two-dimensional unbounded state space. The senders share the same two-dimensional private information and sequentially send messages to the receiver. That is, the second sender can observe what the first sender sent before he/she chooses the message. Sequential communication with several experts is often observable in our life; for example, seeking second opinions, a peer-review process and a debate are categorized into this communication structure. In the paper, we consider whether full information transmission is possible in this setup and, if it is, how and when we achieve it.

The results are as follows. First, we show that Krishna and Morgan's (2001b) successful belief system, *self-serving belief*, which supports a *fully revealing equilibrium*, an equilibrium where the senders' private information is completely transmitted, in one-dimensional unbounded state space models does not work in the two-dimensional model. Their positive result crucially depends on the structure of the one-dimensional models. Second, we suggest a new belief system, *extended selfserving belief*, which supports a fully revealing equilibrium in the two-dimensional model. Finally, we show that the necessary and sufficient condition for the existence of fully revealing equilibria is that the senders' preferences are opposing biased, that is, they are biased in dissimilar directions. Therefore, we can conclude that the directions of preference biases remain important in the twodimensional environment.

It is well known that if the private information is one-dimensional and the state space is unbounded, then full information transmission could be an equilibrium outcome. Krishna and Morgan (2001b) show that if the senders have opposing-biased preferences, then the receiver's self-serving belief supports a fully revealing equilibrium. By taking advantage of the conflicts between the senders, the receiver can make each sender check whether the other sends true messages. Because neither sender has an incentive to lie under the belief, full information transmission is realized as an equilibrium outcome.

However, their useful belief generally does not work in multidimensional environments; that is, one-dimensionality of the state space is necessary for the self-serving belief to work well. In two-dimensional models, the two senders can compromise more easily than in one-dimensional models even if the sender's preferences are opposing biased. The self-serving belief system is fragile in the case of such compromised deviations, and the deviations are omitted in one-dimensional models. Consider, for example, discussion of the tax on alcohol. If the situation is represented by a one-dimensional model, i.e., experts discuss only the total amount of taxes on alcohol, and their preferences are opposing biased, then any compromise is impossible. However, if we consider the same problem in a two-dimensional model, i.e., the experts discuss taxes on both whiskey and wine, then the experts who have opposing-biased preferences on the total amount of taxes could compromise. If the experts agreed with the low tax on wine, they would reach a compromise in terms of a lower wine tax and a higher whiskey tax than the socially optimal levels. One-dimensional models exclude such compromised deviations.

It is also well known that multidimensional cheap talk games have positive results on information transmission. Battaglini (2002) constructs a useful belief system that supports a fully revealing equilibrium. However, his belief system is fragile in the case of sequential communication; if messages are sequential, then his positive result can hold only in the special case. Basically, sequential cases are more difficult than simultaneous cases because of the sequential rationality of the second sender. Moreover, multidimensional models require us to check a number of possible strategies. The literature does not tell us whether full information transmission is possible in multidimensional sequential message models. To pursue the question, this paper extends Krishna and Morgan's (2001b) results to a two-dimensional model.

This paper is structured as follows. In the next subsection, we discuss the related literature. Section 2 defines a two-dimensional sequential message cheap talk game model. In Section 3, we retest the result of Krishna and Morgan (2001b) in the two-dimensional model. We develop a new belief system that works well in the two-dimensional case, and characterize fully revealing equilibria in Section 4. We discuss extensions in Section 5, and conclude the paper in Section 6.

1.1 Related literature

Crawford and Sobel (1982) study a one-dimensional cheap talk game with one sender and one receiver. Their result is that the degree of information transmission depends on the difference between the sender's and the receiver's preferences and, in particular, they show that full information transmission is impossible unless both players' preferences coincide. Following this study, several research streams that consider full information transmission in cheap talk games have developed. The research regarding multiple-sender models is one such stream.

Gilligan and Krehbiel (1989) define the one-dimensional bounded state space, [0, 1], and analyze the situation where the two senders send messages simultaneously in the context of legislation. Krishna and Morgan (2001a) reexamine this problem and show that full information transmission is possible unless the conflict between the players is large. On the other hand, Krishna and Morgan (2001b) analyze the situation where the experts send messages sequentially. Because they also define the one-dimensional bounded state space, [0, 1], they conclude that full information transmission is impossible. However, if the state space is defined as the real line, then the self-serving belief supports a fully revealing equilibrium.¹

Battaglini (2002) defines a two-dimensional unbounded state space, \mathbb{R}^2 , and analyzes simultaneous communication processes. He suggests a belief system that supports a fully revealing equilibrium. Under his belief system, the receiver makes each sender report only one element of the two-dimensional private information; for example, one sender sends a message about the *x*coordinate of the private information, and the other sender sends a message about the *y*-coordinate. By aggregating both messages, the receiver acquires the true information. Battaglini (2002) shows that fully revealing equilibria exist unless the senders' preferences are biased in exactly the same direction. Therefore, he concludes that the important factor for full information transmission is not the degree of conflicts and the bias directions, but the multidimensionality itself.² Ambrus and Takahashi (2008) consider the same situation in bounded state space and point out that Battaglini's full revelation result depends crucially on the unboundedness of the type space. Furthermore, they show the necessary and sufficient condition for full information revelation for any state space.³ Recently, Kawai (2013) extends the sufficiency part of the main result of this paper to an environment with more general preferences.⁴

The paper is different from the above literature in terms of the dimensionality of the state space, communication process, and the properties of belief systems. First, the paper is an extension of Krishna and Morgan's (2001b) one-dimensional unbounded state space model into the two-dimensional environment, and it suggests a new belief system. The extended self-serving belief is more restricted than the original in order to prevent the compromised deviations mentioned above. Second, this paper is also an extension of Battaglini's (2002) simultaneous-communication model into a sequential-communication model. The extended self-serving belief is different from Battaglini's (2002) belief system in the sense that the receiver makes the senders send direct messages, and compares them to obtain true information. Finally, this paper and Kawai (2013) study multidimensional models with sequential communication, but the extended self-serving belief is a complement to Kawai's (2013) belief system in terms of its applicability. That is, Kawai's (2013) belief system is suitable for environments where the receiver is free to choose any action inde-

 $^{^{1}}$ All of the above papers as well as this paper assume that experts are perfectly informed players. Austen-Smith (1993b) analyzes the situation where the experts are imperfectly informed.

 $^{^{2}}$ Chakraborty and Harbaugh (2010) also claim the importance of the multidimensionality by showing the existence of informative equilibria with a state-independent preference sender.

³Austen-Smith (1993a), Battaglini (2004), Levy and Razin (2007), and Ambrus and Lu (2014) study imperfectly informed experts models in multidimensional environments.

⁴Zapechelnyuk (2013), another recent work in this research stream, studies information extraction from multiple experts who could collude in advices.

pendent of the senders' messages. On the other hand, the extended self-serving belief is suitable for environments where the receiver's alternatives are constrained to "recommendations" by the senders.

2 The Model

We consider the following two-dimensional cheap talk game with sequential communication. There are three players: two senders and a receiver. We call the senders *expert 1* and *expert 2*, and the receiver the *decision-maker*.⁵ The experts share private information about the state, which is denoted by a two-dimensional vector. Let $\Theta \equiv \mathbb{R}^2$ be the state space, and let $\theta = (\theta^1, \theta^2) \in \Theta$ be the realized value of the state, which is known to both experts but unknown to the decision-maker. This is the experts' private information.⁶ Note that the state space is unbounded. Let $F(\cdot)$ be a differentiable prior probability distribution function on Θ with density $f(\cdot)$ such that $f(\theta) > 0$ for any $\theta \in \Theta$. Let $S_i \equiv \Theta$ be expert *i*'s message space, for i = 1, 2. Note that each expert uses direct messages and the message sent by expert *i* is denoted by $s_i = (s_i^1, s_i^2) \in S_i$.

Let $Y \equiv \mathbb{R}^2$ be the decision-maker's action set and let $y = (y^1, y^2)$ be the action chosen by the decision-maker. In this model, all players' preferences are different. We describe these differences by parameters x_0, x_1 , and x_2 ; let $x_i = (x_i^1, x_i^2) \in \mathbb{R}^2$ be the expert *i*'s preference bias, and the decision-maker's preference bias, x_0 , is normalized to be (0, 0). Thus, x_i is a measure of how expert *i*'s preference is biased compared with that of the decision-maker. We assume that $x_1 \neq x_2$ and $x_1, x_2 \neq (0, 0)$.

The decision-maker and the experts have von Neumann Morgenstern utility functions, U^D : $Y \times \Theta \to \mathbb{R}, U^{E_i}: Y \times \Theta \times \mathbb{R}^2 \to \mathbb{R}$, respectively, defined as follows:⁷

$$U^{D}(y,\theta) \equiv -\sum_{j=1}^{2} (y^{j} - \theta^{j})^{2}, \qquad (1)$$

$$U^{E_i}(y,\theta,x_i) \equiv -\sum_{j=1}^2 (y^j - (\theta^j + x_i^j))^2.$$
(2)

From (1) and (2), when the value of θ is given, the decision-maker's and the experts' *ideal points* that represent the most preferable actions for each player are θ , $\theta + x_1$ and $\theta + x_2$, respectively. For

 $^{^{5}}$ As a matter of convention, we treat the experts as male and the decision-maker as female throughout this paper.

 $^{^{6}\}mathrm{We}$ assume that both experts are perfectly informed players.

⁷We restrict our attention to the quadratic-loss utility case. This is the usual assumption in the literature on cheap talk games; see Crawford and Sobel (1982), Gilligan and Krehbiel (1989), Krishna and Morgan (2001a,b) and Battaglini (2002).

simplicity, we denote $\theta + x_i$ by O_i . We assume that all information except θ is common knowledge.

It is worth pointing out that the experts' messages do not directly affect all players' payoffs. Thus, this is a cheap talk game. In addition, because we focus on the direct message game with quadratic-loss utility functions, the experts' messages are interpreted as recommendations of the action that the decision-maker should choose.

The timing of the game is as follows. First, nature chooses state θ according to the distribution function $F(\cdot)$, and then both experts observe this value correctly. Second, expert 1 sends a message s_1 , which is dependent on θ . Third, expert 2 sends a message s_2 , which is dependent on both θ and s_1 . Finally, the decision-maker chooses an action y after observing both messages s_1 and s_2 .

Expert 1's pure strategy, $\mu_1 : \Theta \to S_1$, specifies a message s_1 that is sent in the state θ . Expert 2's pure strategy, $\mu_2 : \Theta \times S_1 \to S_2$, specifies a message s_2 that he sends in the state θ after observing expert 1's message s_1 . The decision-maker's pure strategy, $y : S_1 \times S_2 \to Y$, specifies an action y that is chosen after observing both messages s_1 and s_2 . Then, the decision-maker's posterior belief is denoted by $\mathcal{P} : S_1 \times S_2 \to \Delta(\Theta)$. This is a function from a pair of messages to a probability distribution on Θ .

The solution concept is the *perfect Bayesian equilibrium*(hereafter PBE) and we focus on pure strategy equilibria.

Definition 1 A quadruple $(\mu_1^*, \mu_2^*, y^*; \mathcal{P}^*)$ is a PBE if it satisfies the following conditions:

- (i) for any $\theta \in \Theta$, $\mu_1^*(\theta) \in \arg \max_{s_1 \in S_1} U^{E_1}(y^*(s_1, \mu_2^*(\theta, s_1)), \theta, x_1);$
- (*ii*) for any $\theta \in \Theta$ and $s_1 \in S_1$, $\mu_2^*(\theta, s_1) \in \arg \max_{s_2 \in S_2} U^{E_2}(y^*(s_1, s_2), \theta, x_2);$
- (iii) for any $s_1 \in S_1$ and $s_2 \in S_2$, $y^*(s_1, s_2) \in \arg\max_{y' \in Y} \mathbb{E}_{\mathcal{P}^*(\theta|s_1, s_2)}[U^D(y', \theta)];$
- (iv) \mathcal{P}^* is derived using μ_1^* and μ_2^* by Bayes's rule whenever it is possible. Otherwise, \mathcal{P}^* is any probability distribution on Θ .

Because we consider the quadratic-loss utility functions as defined in (1) and (2), expert *i*'s indifference curve is a circle, the center of which is expert *i*'s ideal point, O_i , and the radius of this circle is the norm of the preference bias, $||x_i||$, where $|| \cdot ||$ is the Euclidian norm. We use $I_i(\theta)$ to denote expert *i*'s indifference curve through action $y = \theta$, $R_i(\theta)$ to denote the upper contour set of



Figure 1: Both experts' indifference curves through $y = \theta$

 $I_i(\theta)$, and $P_i(\theta)$ to denote the strict upper contour set of $I_i(\theta)$. In other words:

$$I_i(\theta) \equiv \{ y \in \mathbb{R}^2 \mid U^{E_i}(y, \theta, x_i) = U^{E_i}(\theta, \theta, x_i) \},$$
(3)

$$R_i(\theta) \equiv \{ y \in \mathbb{R}^2 \mid U^{E_i}(y, \theta, x_i) \ge U^{E_i}(\theta, \theta, x_i) \},$$
(4)

$$P_i(\theta) \equiv \{ y \in \mathbb{R}^2 \mid U^{E_i}(y, \theta, x_i) > U^{E_i}(\theta, \theta, x_i) \}.$$
(5)

By the definition of $I_i(\theta)$, $I_1(\theta)$ and $I_2(\theta)$ intersect at least at $y = \theta$, as in Figure 1.

The two-expert situations are divided into the following two cases: *like biases* and *opposing biases*.

Definition 2 The experts have like biases if $x_1 \cdot x_2 > 0$. Otherwise, they have opposing biases; that is, $x_1 \cdot x_2 \leq 0$.

In like-biases cases, the correlation coefficient of the vectors x_1 and x_2 are positive, so we can interpret this to mean that the experts' preferences are biased in similar directions. Geometrically, it is equivalent to $0^{\circ} \leq \gamma < 90^{\circ}$, where γ is the interior angle of x_1 and x_2 . On the other hand, the correlation coefficient is nonpositive in opposing-biases cases, so we can say that the experts' preferences are biased in dissimilar directions. Geometrically, this is equivalent to $90^{\circ} \leq \gamma \leq 180^{\circ}$.

In the following analysis, we focus on a *fully revealing equilibrium* where the private information is fully transmitted. We define a fully revealing equilibrium as follows.

Definition 3 A PBE $(\mu_1^*, \mu_2^*, y^*; \mathcal{P}^*)$ is a fully revealing equilibrium if $y^*(\mu_1^*(\theta), \mu_2^*(\theta, \mu_1^*(\theta))) = \theta$ holds for any $\theta \in \Theta$.

3 Limitations of Self-serving Belief

In this section, we review the *self-serving belief* defined by Krishna and Morgan (2001b), and show that a straightforward application of the belief into the two-dimensional model does not support fully revealing equilibria.

3.1 One-dimensional unbounded state space model

We briefly review the one-dimensional unbounded state space model of Krishna and Morgan (2001b) in this subsection. Hence, we suppose that $\Theta, Y \equiv \mathbb{R}$ and $x_i \in \mathbb{R}$ for i = 1, 2 throughout this subsection. In the one-dimensional model, the opposing-biases cases are defined, without loss of generality, by the cases such that $x_1 < 0$ and $x_2 > 0$. In addition, we define the decision-maker and the experts' one-dimensional quadratic-loss utility functions, $u^D : Y \times \Theta \to \mathbb{R}, u^{E_i} : Y \times \Theta \times \mathbb{R} \to \mathbb{R}$, respectively, by $u^D(y,\theta) = -(y-\theta)^2$ and $u^{E_i}(y,\theta,x_i) = -(y-(\theta+x_i))^2$. The self-serving belief is defined as follows.

Definition 4 Self-serving belief (Krishna and Morgan (2001b))

 (i) A message s₂ from expert 2 is self-serving if the adoption of the recommendation by expert 2 is strictly better for expert 2 than the adoption of the recommendation by expert 1, given that expert 1 sends the true messages. In other words,

$$s_2 \text{ is self-serving if } u^{E_2}(s_2, s_1, x_2) > u^{E_2}(s_1, s_1, x_2).$$
(6)

(ii) The decision-maker has the self-serving belief if the posterior belief $\mathcal{P}(\cdot|s_1, s_2)$ satisfies the following conditions; for any $s_1 \in S_1$ and $s_2 \in S_2$:

$$s_2 \text{ is self-serving } \Rightarrow \mathcal{P}(s_1|s_1, s_2) = 1,$$
 (7)

$$s_2 \text{ is not self-serving } \Rightarrow \mathcal{P}(s_2|s_1, s_2) = 1.$$
 (8)

That is, under the self-serving belief, the decision-maker believes expert 1's message for certain if expert 2's message is self-serving. Otherwise, she believes expert 2's message for certain. This belief system works when the experts have opposing biases.

Proposition 1 (Krishna and Morgan (2001b) footnote 9.)

Consider the one-dimensional unbounded state space model. Suppose that the experts have opposing biases. Then, there exists a fully revealing equilibrium supported by the self-serving equilibrium.



Figure 2: Expert 1 sends the true message.



Figure 3: Expert 1 sends a false message $s'(<\theta)$.

Proof. See Krishna and Morgan (2001b). \blacksquare

The intuition behind the result is described in Figure 2 and 3. Consider an opposing-biases case and suppose that expert 1 sends the true message. Given the message, expert 2 cannot improve his utility by lying because such messages are always self-serving; that is, the decision-maker never believes them. The bold region of Figure 2 is the set of actions that expert 2 can induce. Thus, by sending the true message, expert 1 can induce the first-best action $y = \theta$.

Next, suppose that expert 1 sends a false message, s', which is smaller than, but not far from, θ as described in Figure 3. Given the message s', expert 2 can always send a credible message, $s_2 = s' + 2x_2$. This induces the action $y = s' + 2x_2$, and it is better for expert 2 than $y = \theta$. Because both experts have opposing-biased preferences, $y = s' + 2x_2$ is worse for expert 1 than the first-best action, $y = \theta$. Similarly, if expert 1 sends a false message s'', which is larger than, but not far from, θ , then expert 2 agrees with the message and it is induced. However, it is worse for expert 1 than the first-best action. Thus, expert 1 has no incentive to lie. That is, the decision-maker can make each expert check whether the other expert's message is true, given the self-serving belief. On the equilibrium path, the self-serving belief is consistent with Bayes' rule. Therefore, in the one-dimensional opposing-biases case, the self-serving belief supports a fully revealing equilibrium.



Figure 4: The self-serving belief.

Figure 5: Lemma 1

3.2 Two-dimensional unbounded state space model

Now, we return to the two-dimensional model defined in Section 2, and apply the self-serving belief into the two-dimensional opposing-biases cases, as shown in Figure 4. $I_i(s_1)$ represents expert *i*'s indifference curve that the decision-maker faces when she believes that expert 1's message s_1 is true, and $P_i(s_1)$ and $R_i(s_1)$ are the strict and the weak upper contour sets of $I_i(s_1)$, respectively. We denote $s_1 + x_i$ by O'_i . By using the notation, the self-serving belief is described as follows; for any $s_1 \in S_1$ and $s_2 \in S_2$:

$$s_2 \in P_2(s_1) \Rightarrow \mathcal{P}(s_1|s_1, s_2) = 1,$$
(9)

$$s_2 \notin P_2(s_1) \Rightarrow \mathcal{P}(s_2|s_1, s_2) = 1. \tag{10}$$

Under the self-serving belief, if expert 1 sends $s_1 \neq \theta$, then, by inducing the action $y \in P_2(\theta) \setminus P_2(s_1)$, expert 2 will be better off than if $y = \theta$ is realized. The shaded region in Figure 4 is the set of such actions. If $O_2 \notin P_2(s_1)$, then expert 2's best response is trivial; he sends $s_2 = O_2$. If $O_2 \in P_2(s_1)$, then expert 2 cannot induce his ideal point O_2 under the self-serving belief. The next lemma characterizes the most preferred action that expert 2 can induce when $O_2 \in P_2(s_1)$. By this lemma, we can insist that expert 2's best response exists on the half-line through O'_2 and O_2 , the initial point of which is O'_2 .

Lemma 1 Fix s_1 such that $O_2 \in P_2(s_1)$, and let y^* be the closest point on $I_2(s_1)$ to expert 2's ideal point O_2 . Then, y^* is the intersection of $I_2(s_1)$ and the half-line through O'_2 and O_2 , the initial point of which is O'_2 .

Proof. All proofs are in Appendix A. \blacksquare



Figure 6: Expert 1's profitable deviation

Consider opposing-biases cases with $90^{\circ} \leq \gamma < 180^{\circ}$. In these cases, $P_1(\theta) \cap P_2(\theta)$, a set of actions that both experts strictly prefer to action $y = \theta$, is nonempty. Hence, the experts can induce an action in $P_1(\theta) \cap P_2(\theta)$ by deviating from truth-telling because the self-serving belief is fragile in the face of such compromised deviations, which is explained as follows. Suppose that expert 1 sends message s_1 such that $\overrightarrow{\theta s_1}$ is parallel to the line through O_1 and O_2 , and $||\theta - s_1|| = \epsilon > 0$ where ϵ is small enough, as shown in Figure 6. Then, O'_2 exists on the line through O_1 and O_2 . By Lemma 1, expert 2's best response is sending $s_2 = s^*$, the intersection of $I_2(s_1)$ and the half-line O'_2O_2 , starting at O'_2 . Under the self-serving belief, it is not self-serving, so action $y = s^*$ is induced. However, because the deviation is so small, action $y = s^*$ exists in $P_1(\theta) \cap P_2(\theta)$ as shown in Figure 6. Because the experts strictly prefer the induced action to $y = \theta$, expert 1 has an incentive to deviate from truth-telling and expert 2 endorses expert 1's deviation. Therefore, we can say that, in the two-dimensional model, the self-serving belief can support fully revealing equilibria only when the experts have perfectly opposing biases.

Proposition 2 Consider the two-dimensional unbounded state space model. Then, there exists a fully revealing equilibrium supported by the self-serving belief if and only if $P_1(\theta) \cap P_2(\theta) = \emptyset$.

Geometrically, the necessary and sufficient condition is that the experts' indifference curves $I_1(\theta)$ and $I_2(\theta)$ circumscribe at $y = \theta$; that is, $\gamma = 180^\circ$, the perfectly opposing-biases case. In the twodimensional model, there are other "intermediate" opposing-biases cases such that $90^\circ \leq \gamma < 180^\circ$. The self-serving belief is fragile to these intermediate cases. However, because of the structure of the models, we only focus on the perfectly like-biases and the perfectly opposing-biases cases in onedimensional models. In other words, the intermediate cases are ignored, so the self-serving belief system is sufficient to support fully revealing equilibria in one-dimensional models. Therefore, we can conclude that the positive result in the one-dimensional model crucially depends on the one-dimensional structure.

This raises a new question as to whether fully revealing equilibria exist when $P_1(\theta) \cap P_2(\theta) \neq \emptyset$. We find the following positive result in the literature.

Proposition 3 (Battaglini (2002) p.1395)

Consider the two-dimensional unbounded state space model. Then, there exists a fully revealing equilibrium supported by Battaglini's belief system if and only if $x_1 \cdot x_2 = 0$.

Proof. See Battaglini (2002). ■

The case of $x_1 \cdot x_2 = 0$ is equivalent to $\gamma = 90^{\circ}$ and it is an opposing-biases case. However, this belief system is also fragile in the face of the other intermediate cases.

In summary so far, we have already known that there exists a fully revealing equilibrium if the experts have either (i) perfectly opposing biases, i.e., $\gamma = 180^{\circ}$, or (ii) orthogonal biases, i.e., $\gamma = 90^{\circ}$. On the other hand, the literature does not answer the question of whether there exists a fully revealing equilibrium when the experts have (i) intermediate opposing biases, i.e., $90^{\circ} < \gamma < 180^{\circ}$ or (ii) like biases, i.e., $0^{\circ} \le \gamma < 90^{\circ}$.

4 Extended Self-serving Belief and Fully Revealing Equilibria

This section studies the open questions discussed in the last paragraph. First, we suggest a new belief system, *extended self-serving belief*, and show that there exists a fully revealing equilibrium supported by the new belief system if the experts have opposing biases. Second, we show that there exist no fully revealing equilibria in the like-biases cases. That is, the existence of opposing-biased preferences is the necessary and sufficient condition for full information transmission.

Let us introduce some notation. Given expert 1's message s_1 , let \hat{s}_1 be the other intersection of $I_1(s_1)$ and $I_2(s_1)$. Consider two tangents of $I_1(s_1)$ at s_1 and \hat{s}_1 , and let O_{s_1} be the intersection of the tangents. Let $\mathcal{T}(s_1)$ be the interior of the cone, the vertex of which is O_{s_1} and the sides are the tangents of $I_1(s_1)$ at s_1 and \hat{s}_1 . The extended self-serving belief is defined as follows.

Definition 5 Extended self-serving belief.⁸

The decision-maker has the extended self-serving belief if the posterior belief $\mathcal{P}(\cdot|s_1, s_2)$ satisfies the

⁸I am very grateful to Nozomu Muto for suggesting this criterion. Originally, I used a stronger criterion in the sense that more conditions are needed to construct a fully revealing equilibrium.



Figure 7: Extended self-serving belief



Figure 8: Preventing the deviation

following conditions; for any $s_1 \in S_1$ and $s_2 \in S_2$:

s

$$_{2} \in P_{2}(s_{1}) \cup \mathcal{T}(s_{1}) \Rightarrow \mathcal{P}(s_{1}|s_{1},s_{2}) = 1,$$

$$(11)$$

$$s_2 \notin P_2(s_1) \cup \mathcal{T}(s_1) \Rightarrow \mathcal{P}(s_2|s_1, s_2) = 1.$$

$$(12)$$

Expert 2's messages in the shaded region or on the bold line in Figure 7 are credible under the extended self-serving belief. It restricts the set of credible messages for expert 2 more than the original. Under the extended self-serving belief, the decision-maker believes expert 1's message for certain if expert 2's message is $s_2 \in P_2(s_1) \cup \mathcal{T}(s_1)$. Otherwise, she believes expert 2's message for certain.

The extended self-serving belief is interpreted as follows.⁹ We define $s_1^*(s_1, s_2) \equiv \arg\min_{s \in \{s_1, \hat{s}_1\}} ||s_2 - s||$, given s_1 and s_2 . We assume that there is a decision-maker who believes one or the other of experts' messages for certain. First, the decision-maker believes s_2 if $s_1 = s_2$, that is, expert 2 endorses expert 1. In addition, the decision-maker believes $s_2 = \hat{s}_1$ for certain; given the first point, expert 2 has no incentive to send $s_2 = \hat{s}_1$ if $s_1 = \theta$, i.e., expert 1 tells the truth. Because expert 2 is indifferent between $y = s_1$ and $y = \hat{s}_1$ as long as $s_1 = \theta$, the decision-maker knows that expert 2 has no incentive to send $s_2 = \hat{s}_1$ if expert 1 tell the truth. Hence, the decision-maker can infer that expert 1 misreports when she observes $s_2 = \hat{s}_1$, and never believes such messages by expert 1. Then, the decision-maker believes $s_2 \neq s_1, \hat{s}_1$ if and only if (i) it is not self-serving and (ii) the direction that expert 2 must show that (i) his message is not self-serving as in the original, and (ii) the direction of a deviation from $s_1^*, \overline{s_1^*s_2}$, is "not like" the direction in which expert 1 would

⁹I really appreciate the advice of an anonymous referee who suggested this interpretation.



Figure 9: Examples of successful deviations

like the decision-maker to move from s_1^* , $\overrightarrow{s_1^*O_1'}$, i.e., the interior angle of $\overrightarrow{s_1^*s_2}$ between $\overrightarrow{s_1^*O_1'}$ must be obtuse. The latter means that the vector $\overrightarrow{s_1^*s_2}$ never passes through $P_1(s_1)$; the complement of $\mathcal{T}(s_1)$ is the set of such messages by expert 2. Therefore, the decision-maker believes s_2 if and only if $s_2 \notin P(s_1) \cup \mathcal{T}(s_1)$.

We return to the problematic deviation for the self-serving belief mentioned in the last section, and demonstrate how the extended self-serving belief prevents it. Under the self-serving belief, expert 1 can induce action $y = s^*$ in Figure 8 because, given s_1 , expert 2's best response, $s_2 = s^*$, is credible for the decision-maker. However, under the extended self-serving belief, s_2 is not credible. Given s_1 , expert 2's best response is either $s_2 = s_1$ or $s_2 = \hat{s}_1$. The decision-maker adopts expert 2's message, but the induced action $y = s_1$ or \hat{s}_1 is worse than $y = \theta$ for expert 1. Therefore, the extended self-serving belief can prevent the deviation.

We can show that, as long as the experts have opposing biases, any deviation from the truth never improves expert 1's payoff under the extended self-serving belief, so he has no incentive to deviate from $s_1 = \theta$.¹⁰ Expert 2's best response given $s_1 = \theta$ is the endorsement of it, $s_2 = \theta$. Hence, at any state, the experts' private information is completely transmitted to the decision-maker. That is, we have a positive answer to the first question proposed in the last section; there also exists a fully revealing equilibrium in the intermediate opposing-biases cases, i.e., $90^{\circ} < \gamma < 180^{\circ}$.

It is worthwhile to mention why it is insufficient to exclude only $P_1(s_1) \cup P_2(s_1)$. It may seem that when the experts have opposing biases, excluding $P_1(s_1)$ is sufficient to prevent deviations that

¹⁰The formal proof is in the Appendix A.



Figure 10: Lemma 2

induce better actions for expert 1. However, this is not correct; unboundedness of the excluded region is necessary. Consider the following belief system; the decision-maker believes a message s_2 if and only if $s_2 \notin P_1(s_1) \cup P_2(s_1)$. This belief system does not always support a fully revealing equilibrium even if the experts have opposing biases.

If $||x_1|| > ||x_2||$, then expert 1 can deviate such that $P_1(s_1)$ includes $P_2(\theta)$, as shown in Figure 9-(a). We face the same problem that the original self-serving belief faces; given such s_1 , expert 2's best response is $s_2 = s'$, which is on $I_1(s_1)$, and the decision-maker believes it. Therefore, it is a profitable deviation for expert 1. Moreover, if $||x_1|| \le ||x_2||$ but $||x_2|| - ||x_1||$ is not large enough, then the same problem also occurs; expert 1 can induce action $y = s' \in P_1(\theta)$, as shown in Figure 9-(b). In addition to opposing biases, we need $||x_2 - x_1|| \ge 2||x_1||$ to support a fully revealing equilibrium under this belief system.¹¹ As long as the excluded region is bounded in the direction of O'_1 , we face the same problem, so we need additional conditions regarding the norms of the preference biases for supporting fully revealing equilibria. Thus, we have to exclude the unbounded region, like $\mathcal{T}(s_1)$, to avoid such constraints.

Next, we consider the second question; do fully revealing equilibria exist in the like-biases cases, i.e., $0^{\circ} \leq \gamma < 90^{\circ}$? Our answer is negative; there exists no fully revealing equilibrium in like-biases cases. The last part of this section demonstrates why full information transmission is impossible when the experts have like biases. Consider the line segment connecting θ with O_2 , and call it the line of endorsement at θ .¹² As Lemma 1 shows, at any state θ , if expert 1 deviates to some $s_1 \neq \theta$ that lies on the line of endorsement at θ , then expert 2 always endorses the deviation; that is, $s_2 = s_1$ is the unique best responses of expert 2. In other words, by deviating in the direction of expert 2's preference bias, x_2 , expert 1 can force expert 2 to endorse the deviation. The next

¹¹The formal proof is available upon request.

 $^{^{12}}$ I thank the anonymous referee who suggested this notion.



Figure 11: Impossibility in like-biases cases

lemma gives an equivalent condition for bias relations.

Lemma 2 Consider the two-dimensional unbounded state space model. Then, the experts have opposing biases if and only if for any state θ , the intersection of $P_1(\theta)$ and the line of endorsement at θ is empty.¹³

The impossibility of full information transmission in like-biases cases comes from the nonemptiness of the intersection of $P_1(\theta)$ and the line of endorsement. That is, expert 1 is strictly better off by deviating to some point on the intersection of $P_1(\theta)$ and the line of endorsement, and no belief system ever prevents such deviations. Intuitively, consider a like-biases case in Figure 11. We suppose, in contrast, that there exists a fully revealing equilibrium in this case. If expert 1 pretends to be state θ' when the true state is θ , then endorsing this deviation is the unique best response of expert 2 because θ' lies in the line of endorsement at θ . Hence, action $y = \theta'$ is induced, and expert 1 strictly prefers it to $y = \theta$, which is a contradiction. Under any belief system, expert 2 never contests such deviations by expert 1, so we can conclude that there exists no fully revealing equilibrium in like-biases cases. Therefore, fully revealing equilibria in the two-dimensional unbounded state space model are characterized as follows.

¹³As long as we consider the quadratic-loss utility case, the same property holds for the line segment connecting O_2 and $\hat{\theta}$, which is the other intersection of $I_1(\theta)$ and $I_2(\theta)$.

Proposition 4 Consider the two-dimensional unbounded state space model. Then, there exists a fully revealing equilibrium if and only if the experts have opposing biases.

As shown in Lemma 2, the condition that the experts have opposing biases is equivalent to the condition that the intersection of $P_1(\theta)$ and the line of endorsement at θ is empty, and this emptiness is crucial for the existence of fully revealing equilibria, as shown in Proposition 4. That is, properties of the intersection of $P_1(\theta)$ and the line of endorsement at θ provides an exact analogy with the definitions of Krishna and Morgan's (2001b) one-dimensional opposing/like biases. Therefore, this point makes clear the connection between one-dimensional and two-dimensional models, even though these are often discussed as somehow fundamentally different models in the literature.

5 Discussion and Extensions

5.1 Collusion

Sequential communication can be regarded as a situation where experts in a committee can collude before advising a decision-maker. That is, expert 2 can choose whether to collude with expert 1 before sending his recommendation. Hence, the fully revealing equilibrium supported by the extended self-serving belief can be said to robust with respect to this kind of collusion.

Zapechelnyuk (2013) studies collusion among experts in a committee under the framework of cheap talk games; there are n experts who share the same multidimensional private information, and the experts engage in bargaining before giving recommendations to a decision-maker. Instead of specifying the bargaining procedure, Zapechelnyuk (2013) imposes axioms that any bargaining solution must satisfy. In his environment, the decision-maker can elicit full information if and only if the outcome induced by a fully revealing equilibrium is not Pareto dominated by the experts in the committee.

This paper is a complement of Zapechelnyuk (2013). As Proposition 4 has shown, in our environment, the decision-maker can obtain full information even if the outcome induced by a fully revealing equilibrium is Pareto dominated by the experts as long as they have opposing biases. The discrepancy of the results in the two papers comes from the differences in how the experts deviate. Zapechelnyuk (2013) mainly focuses on *opting-out deviations*. That is, if expert $i \in \{1, 2, ..., n\}$ deviates, then the other n - 1 experts have chances to react this deviation. However, in our setup, expert 2 has a chance to react to expert 1's deviations, but expert 1 cannot react to expert 2's deviations. In other words, Zapechelnyuk (2013) focuses on the scenario of full opting-out in the sense that all experts can react to the other's deviations. On the other hand, our bargaining procedure is a partial opting-out in the sense that only one of the experts can react to the other's deviations.¹⁴

5.2 N(>2)-dimensional models.

The results can be extended to N(> 2)-dimensional models as long as quadratic-loss utility functions are assumed. We can easily define the N-dimensional extended self-serving belief in a similar way to the two-dimensional model. The construction of the fully revealing equilibrium depends on the distance between the experts' true indifference curves and the illusionary ones. The assumption of two-dimensionality is not essential for this construction. Thus, we can obtain the same results in N-dimensional unbounded state space models. Moreover, because the decision-maker can elicit true information by taking advantage of the conflict between two experts, this means that two experts are sufficient for the existence of a fully revealing equilibrium even in N-dimensional models.

5.3 n(>2)-expert models

We consider a situation where there are n(>2) experts who share the same two-dimensional private information, and send messages sequentially. The sufficiency of Proposition 4 can be easily extended to the *n*-expert model. Like and opposing biases in the *n*-expert model are defined as follows. The experts are said to have *like biases* if $x_i \cdot x_j > 0$ for any experts *i* and *j*. Otherwise, the experts are said to have *opposing biases*. If the experts have opposing biases, then we can find a fully revealing equilibrium that is essentially equivalent to that in the two-expert model. That is, because there exists a pair of experts *i* and *j* such that $x_i \cdot x_j \leq 0$, the decision-maker can elicit full information by caring only about the messages from experts *i* and *j* under the extended self-serving belief, and ignoring the other messages.

However, an extension of the necessary part of Proposition 4 is not straightforward. The difficulty is specifying how each expert reacts to the predecessors' behaviors without specifying the decision-maker's belief. This problem can be avoided in the two-expert model by considering a deviation along the line of endorsement; expert 2 endorses such a deviation whatever the decision-maker's belief is. However, in the *n*-expert model, because of the sequential rationality of the subsequent experts, finding a deviation that every expert endorses without specifying the decision-maker's belief is nontrivial, even if the experts have like biases. We have the following partial extension. Let $K_i(\theta, \theta') \equiv \{y \in Y | U^{E_i}(y, \theta, x_i) \geq U^{E_i}(\theta', \theta, x_i)\}.$

¹⁴I thank Andriy Zapechelnyuk for conversations about it.



Figure 12: An example of Proposition 5

Proposition 5 Consider the three-expert model, and suppose that for any $\theta \in \Theta$, there exists $\theta' \neq \theta \in \Theta$ such that (i) θ' lies on the line of endorsement of expert 3 at θ and (ii) $K_2(\theta, \theta') \subset P_1(\theta)$. Then, there exists no fully revealing equilibrium.

Intuitively, the conditions in Proposition 5 mean that (i) each bias has the similar magnitude, and (ii) the interior angles of the biases are sufficiently small as shown in Figure 12. Hence, if the conditions are satisfied, then the experts have like biases.¹⁵ Two open questions remain. First, we conjecture that a fully revealing equilibrium never exists in the general *n*-expert model under the similar conditions in Proposition 5. However, we need nontrivial modifications on the conditions and the proof. Second, even in the three-expert model, we do not have the answer when the experts are like biases but the conditions in Proposition 5 are not satisfied.

5.4 Mixed strategies

The necessary condition for the existence of fully revealing equilibrium does not change even if we adopt mixed strategies. Ambrus and Takahashi (2008) show that in simultaneous communication, allowing mixed strategies by the experts could generate a fully revealing equilibrium when there is no fully revealing equilibrium in pure strategies. That is, stochastic outcomes generated by mixed strategies prevent the experts from deviations. However, this logic does not hold in sequential communication because the decision problem that expert 2 faces is equivalent to that in pure strategies even if mixed strategies are allowed. In other words, expert 2's best response could depend on each realized message in sequential communication even if expert 1 undertakes mixed strategies. Because expert 2 is not forced to face stochastic outcomes in his decision making, the same logic used in pure strategies can be applied for showing the necessary condition. There exists no fully revealing equilibrium as long as the experts have like biases, even if the players are allowed

 $^{^{15}\}mathrm{The}$ formal proof is in the Appendix C.

to undertake mixed strategies.¹⁶

5.5 Noisy information

In this subsection, we consider a noisy-information environment by modifying the baseline model as follows. We assume that each expert could not observe the correct state with positive probability. Let $\sigma_i \in \Sigma \equiv \{\theta, \phi\}$ represent expert *i*'s private observation about the state; that is, expert *i* observes nothing (i.e., $\sigma_i = \phi$) with probability $\epsilon_i \in [0, 1)$, and he observes the correct state (i.e., $\sigma_i = \theta$) with probability $1 - \epsilon_i$. We say that expert *i* is *perfectly informed* if $\epsilon_i = 0$; that is, the perfectly informed expert always observes the correct state, as in the baseline model. Because the experts' strategies depend on their observations, the definition of fully revealing equilibrium is modified as follows; we say that a PBE $(\mu_1^*, \mu_2^*, y^*; \mathcal{P}_1^*, \mathcal{P}_2^*, \mathcal{P}^*)$ is a fully revealing equilibrium if for any $\theta \in \Theta$ and $\sigma_1, \sigma_2 \in \Sigma$:

$$y^*(\mu_1^*(\sigma_1), \mu_2^*(\sigma_2, \mu_1^*(\sigma_1))) = \begin{cases} \theta & \text{if } (\sigma_1, \sigma_2) \neq (\phi, \phi) \\ \mathbb{E}[\theta] & \text{otherwise} \end{cases}$$
(13)

where \mathcal{P}_i^* represents expert *i*'s posterior belief, and $\mathbb{E}[\theta]$ represents the expectation of the state given the prior probability distribution. Except for this modification, the setup is identical to that in the baseline model. The following proposition summarizes the results.

Proposition 6 Consider the two-dimensional unbounded state space model with noisy observations.

- (i) If $x_1 \cdot x_2 < 0$, $\epsilon_1 = 0$ and ϵ_2 is sufficiently small, then there exists a fully revealing equilibrium.
- (ii) If $\epsilon_1 > 0$, then there exists no fully revealing equilibrium.

The first part of Proposition 6 means that the fully revealing equilibrium supported by the extended self-serving belief is robust to noise such that only expert 2 observes nothing with small probability. However, the second part of this proposition says that once expert 1 becomes imperfectly informed, there never exists a fully revealing equilibrium even though the probability of expert 1 observing nothing is sufficiently small. In other words, independent of the decision-maker's belief, expert 2 has an incentive to deviate when expert 1 observes nothing.

There are two remarks. First, the perfect informativeness of expert 1 is the necessary condition for the existence of fully revealing equilibrium in this noisy environment. That is, the small im-

 $^{^{16}\}mathrm{The}$ formal statement is in the Appendix C.

perfectness of expert 2 is irrelevant to full information transmission as long as expert 1 is perfectly informed, but the imperfectness of expert 1 is fatal. This difference arises because of sequential communication. The case of $(\sigma_1, \sigma_2) = (\theta, \phi)$ is essentially equivalent to the case of $(\sigma_1, \sigma_2) = (\theta, \theta)$ in the sense that expert 2 can learn the true state by observing expert 1's message in a fully revealing equilibrium. In other words, in a fully revealing equilibrium, expert 2 has an additional chance to learn the true state before sending a message. Because we can guarantee the "truth-telling" of expert 1 as long as the imperfectness of expert 2 is sufficiently small, the fully revealing result in the case of $(\sigma_1, \sigma_2) = (\theta, \theta)$ can be replicated in the case of $(\sigma_1, \sigma_2) = (\theta, \phi)$. However, the case of $(\sigma_1, \sigma_2) = (\phi, \theta)$ is different from above cases in the sense of informativeness of the experts; that is, expert 1 has no chance to learn the true state if he observes nothing. Because, in this scenario, the decision-maker then has to completely rely on the expert 2's message for learning the true state, the decision-maker cannot prevent expert 2 from "lying" whatever belief she has, like the one-sender model. This is the reason why the perfect informativeness of expert 1 is crucial to the result.

Second, this fully revealing equilibrium is sensitive to other types of noise. For example, suppose that the observation of expert *i* is given by $\sigma_i \equiv \theta + \epsilon_i$, where ϵ_i follows a normal distribution with zero mean, and ϵ_1 and ϵ_2 are independent.¹⁷ In this environment, disagreements among reports could happen with positive probability even if the experts truthfully report their own observations. Therefore, because the extended self-serving belief is discontinuous in messages, this belief system does not work well under such an information structure.

5.6 General preferences

The sufficiency part of Proposition 4 is extended by Kawai (2013) to an environment with more general preferences. Kawai (2013) assumes that (i) for any $\theta \in \Theta$, U^i is continuous and quasiconcave in $y \in Y$, and single-peaked with ideal point O_i , (ii) for any $\theta \in \Theta$, $y \in Y$ and $z \in \mathbb{R}^N$, $U^i(y,\theta,x_i) = U^i(y+z,\theta+z,x_i)$ holds, (iii) for any $\theta \in \Theta$, there exists $\beta(\theta) > 1$ such that $U^{E_2}(\theta,\theta,x_2) = U^{E_2}(\theta + \beta(\theta)x_2,\theta,x_2)$ holds, and (iv) for any $\theta \in \Theta$, $U^i(\theta + \gamma x_i + \tilde{\gamma}\tilde{x}_i,\theta)$ is strictly decreasing in $|\tilde{\gamma}|$ where \tilde{x}_i is a unit vector such that $x_i \cdot \tilde{x}_i = 0$. For easy reference, preferences satisfying the above assumptions are called as *general preferences*.

Let us introduce additional notation. Let $H_i(y;\theta)$ be a hyperplane that is tangent to $I_i(\theta)$ at action y. Define $H_i^+(\theta) \equiv \{y \in Y | (y - \theta) \cdot x_i > 0\}$ and $H_i^-(\theta) \equiv \{y \in Y | (y - \theta) \cdot x_i < 0\}$, which are the half-spaces divided by hyperplane $H_i(\theta;\theta)$. Let $g(s_1, s_2)$ be the foot of perpendicular from

¹⁷This noisy information structure is studied by Battaglini (2004) and Ambrus and Lu (2014).

 s_2 to hyperplane $H_2(s_1; s_1)$. Define $\tilde{s}_1 \equiv s_1 + \beta x_2$.¹⁸ Kawai (2013) proposes the following belief system; for easy reference, we call the belief system the *open self-serving belief*.

Definition 6 Open self-serving belief. (Kawai (2013))

The decision-maker has the open self-serving belief if the posterior belief $\mathcal{P}(\cdot|s_1, s_2)$ satisfies the following conditions; for any $s_1 \in S_1$ and $s_2 \in S_2$:

$$s_2 \notin H_2^+(s_1) \cap H_2^-(\tilde{s}_1) \Rightarrow \mathcal{P}(s_2|s_1, s_2) = 1$$
 (14)

$$s_2 \in H_2^+(s_1) \cap H_2^-(\tilde{s}_1) \quad \Rightarrow \quad \mathcal{P}(g(s_1, s_2)|s_1, s_2) = 1$$

$$\tag{15}$$

Proposition 7 (Theorem 1 of Kawai (2013))

Consider the multidimensional unbounded state space model with general preferences. Then, there exists a fully revealing equilibrium supported by the open self-serving belief if and only if $x_1 \cdot x_2 \leq 0$.

The main difference between the extended and open self-serving beliefs is the response to selfserving messages. Under the extended self-serving belief, the decision-maker believes that expert 1's message is correct for certain. Under the open self-serving belief, on the other hand, the decisionmaker may believe neither message; she believes that state $\theta = g(s_1, s_2)$ realizes for certain. In other words, if expert 2 wants to rebut expert 1, then he must send non-self-serving messages under the extended self-serving belief. However, expert 2 can rebut even by sending self-serving messages under the open self-serving belief. As a result, expert 1's inducible action set is restricted to the line segment connecting θ and $\theta + \beta x_2$, and then $x_1 \cdot x_2 \leq 0$ guarantees full information transmission.

Because of the above difference, the extended and open self-serving beliefs have different applicability. We assume the quadratic-loss preferences for easy comparison, and consider a situation where the decision-maker's alternatives could be constrained depending on the experts' messages, like legislative processes.¹⁹ The open self-serving belief is appropriate in environments where the decision-maker is free to undertake any action irrelevant to the messages.²⁰ However, if the decisionmaker is constrained such that she has to adopt either one of the "recommendations" by the experts, then the extended self-serving belief seems more appropriate; the open self-serving belief is no longer valid because the response to self-serving messages violates that constraint. Therefore, we can conclude that the extended self-serving belief is a complement to the open self-serving belief in terms

¹⁸By assumption (ii), $\beta(\theta') = \beta(\theta'')$ for any $\theta', \theta'' \in \Theta$. Then, it is simply represented by β .

¹⁹In the literature of legislature under asymmetric information, see Gilligan and Krehbiel (1989) and Krishna and Morgan (2001a), legislative rules without any restriction are called *open rules*, and restricted rules such that the legislature has to adopt one of the recommended proposals by the experts are called *modified rules*.

²⁰It is obvious that the extended self-serving belief is also valid in those environments.

of its applicability.

Although Kawai (2013) generalizes the sufficiency part of Proposition 4, the generalization of the necessary part is limited. Because his statement depends on the particular belief system, the possibility of full information transmission under different belief system is still an open question.

5.7 Bounded state space

The unboundedness of the state space is also a crucial assumption for our results. If we consider a bounded type space, the extended self-serving belief may not imply a fully revealing equilibrium. This is consistent with Krishna and Morgan (2001b) and Ambrus and Takahashi (2008).

6 Conclusion

In this paper, we have studied a sequential cheap talk game with two-dimensional unbounded state space. We have two main findings; first, the self-serving belief suggested by Krishna and Morgan (2001b) generally does not support fully revealing equilibria in the two-dimensional environment; it works if and only if the experts have perfectly opposing biases. In the two-dimensional environment, there exist outcomes where both experts are strictly better off than the first-best outcome even if the experts' preferences are biased in "not like" directions. The self-serving belief is fragile in the face of such "intermediate" opposing-biases cases.

Second, we characterize the necessary and sufficient condition for the existence of fully revealing equilibria in the two-dimensional environment, which is that the experts have opposing biases. As the intersection of $P_1(\theta)$ and the line of endorsement at θ is empty in opposing-biases cases, an appropriate belief system can support fully revealing equilibria. We suggest the extended selfserving belief, under which the decision-maker believes expert 2's messages if and only if (i) it is not self-serving, and (ii) it "contests" expert 1's message, that is, the direction in which expert 2 recommends the decision-maker to move never benefits expert 1. On the other hand, if the experts have like biases, then the intersection of $P_1(\theta)$ and the line of endorsement at θ is not empty. Expert 1's deviation to some point on the intersection makes expert 1 strictly better off, and expert 2 always endorses such deviations under any belief system. Therefore, full information transmission is impossible.

Appendix A: Proofs

First, we define the following notation. For any $a, b \in Y$, let $L(a, b) \equiv \{y \in Y | \exists \alpha \in \mathbb{R} \text{ s.t } \overrightarrow{ay} = \alpha \overrightarrow{ab}\},$ $L^+(a, b) \equiv \{y \in Y | \exists \alpha > 0 \text{ s.t } \overrightarrow{ay} = \alpha \overrightarrow{ab}\}, \text{ and } \overline{L}(a, b) \equiv \{y \in Y | \exists \alpha \in [0, 1] \text{ s.t } \overrightarrow{ay} = \alpha \overrightarrow{ab}\}.$ Geometrically, L(a, b) represents the line $ab, L^+(a, b)$ represents the half-line ab, the initial point of which is a, and $\overline{L}(a, b)$ represents the segment ab. Let $\hat{\theta}$ be the other intersection of $I_1(\theta)$ and $I_2(\theta).$

Proof of Lemma 1

Without loss of generality, assume that $O_2 = (0,0)$ and $O'_2 = (a,0)$, where $0 < a < ||x_2||$. Take an arbitrary point $(b,c) \in I_2(s_1)$. Then:

$$(b-a)^2 + c^2 = ||x_2||^2 \iff b^2 + c^2 = ||x_2||^2 - a^2 + 2ab.$$
 (A.1)

If we let $f \equiv b^2 + c^2$, then f is the square of the distance from O_2 to the point (b,c). Thus, $f = ||x_2||^2 - a^2 + 2ab$, and f is minimized when b is minimized. By construction, $b = a - ||x_2||$. Because $\overrightarrow{O'_2O_2} = (-a,0)$ and $\overrightarrow{O'_2y^*} = (-||x_2||,0)$, $\overrightarrow{O'_2y^*} = \frac{||x_2||}{a}\overrightarrow{O'_2O_2}$. Then, $y^* \in L^+(O'_2,O_2)$. Therefore, $y^* \in L^+(O'_2,O_2) \cap I_2(s_1)$. That is, the closest point y^* is the intersection of $I_2(s_1)$ and the half-line through O'_2 and O_2 , the initial point of which is O'_2 .

Proof of Proposition 2

(Sufficiency) Suppose that $P_1(\theta) \cap P_2(\theta) = \emptyset$. In other words, $R_1(\theta) \cap R_2(\theta) = \{\theta\}$ and neither $R_1(\theta) \subset R_2(\theta)$ nor $R_2(\theta) \subset R_1(\theta)$. If $s_1 = \theta$, then, from the self-serving belief system, expert 2's best response is $s_2 = \theta$. If $s_1 \neq \theta$, then $P_2(\theta) \setminus P_2(s_1) \neq \emptyset$. Because $P_1(\theta) \cap P_2(\theta) = \emptyset$ from the hypothesis, $y \in P_2(\theta)$ and $y \notin P_1(\theta)$ for all $y \in P_2(\theta) \setminus P_2(s_1)$. Hence, for such action y, $U^{E_1}(\theta, \theta, x_1) \geq U^{E_1}(y, \theta, x_1)$. From the self-serving belief system, if $s_1 \neq \theta$, then expert 2 induces the action $y \in P_2(\theta) \setminus P_2(s_1)$. Because expert 1 cannot strictly improve his utility by sending false messages, he has no incentive to lie. Therefore, on the equilibrium path, both experts send messages involving the truth, and the self-serving belief is consistent with Bayes' rule. This is a fully revealing equilibrium.

(Necessity) By definition, $I_1(\theta) \cap I_2(\theta) \neq \emptyset$. Then, there are the following three cases: (i) $I_1(\theta) \cap I_2(\theta) = \{\theta, \hat{\theta}\}$, where $\hat{\theta} \neq \theta$, (ii) $I_1(\theta) \cap I_2(\theta) = \{\theta\}$ and either $R_1(\theta) \subset R_2(\theta)$ or $R_2(\theta) \subset R_1(\theta)$, and (iii) $I_1(\theta) \cap I_2(\theta) = \{\theta\}$ and neither $R_1(\theta) \subset R_2(\theta)$ nor $R_2(\theta) \subset R_1(\theta)$.

(i) case: Let $A \equiv \{y \in Y | L(O_1, O_2) \cap R_1(\theta) \cap R_2(\theta)\}$ and $D \equiv \text{diam } A$. Because the set A is



Figure A.1: Proposition 2

compact, $\exists a, b \in A$ such that ||a - b|| = D.

Case 1: $O_1 \notin P_2(\theta)$. Suppose that expert 1 sends the message $s_1 \neq \theta$ such that $\overrightarrow{\theta s_1} = \epsilon \overrightarrow{O_1O_2}$, where ϵ is such that $0 < ||\epsilon \overrightarrow{O_1O_2}|| < D$ and $O_2 \in P_2(s_1)$. Then, $\overrightarrow{O_2O_2} = \epsilon \overrightarrow{O_1O_2}$. By Lemma 1, $y^* \in L(O_1, O_2)$. Without loss of generality, assume that $a = y^* - \epsilon \overrightarrow{O_1O_2}$. Suppose, in contrast, that $y^* \notin P_1(\theta)$. That is $||O_1 - y^*|| \ge ||x_1||$. Because $O_1 \notin P_2(\theta)$, $||x_1|| = ||O_1 - a|| + D$. Hence:

$$||x_1|| \leq ||O_1 - y^*|| \leq ||O_1 - a|| + ||a - y^*|| = ||O_1 - a|| + ||\epsilon \overrightarrow{O_1 O_2}||$$

 $< ||O_1 - a|| + D = ||x_1||, \text{a contradiction.}$ (A.2)

Then, $y^* \in P_1(\theta)$ must hold. That is, expert 1 has an incentive to lie. Therefore, the self-serving belief does not support fully revealing equilibria in this case.

Case 2: $O_1 \in P_2(\theta)$. Without loss of generality, assume that $||O_1 - a|| \leq ||O_1 - b||$. Consider the following message s_1 such that $\overrightarrow{\theta s_1} = \beta \overrightarrow{O_1 O_2}$, where $\beta = ||O_1 - a||$. From Lemma 1, $y^* \in L(O_1, O_2)$. Note that this is the most preferred action for expert 2 that he can induce. By construction, $y^* = a - \overrightarrow{O_1 a} = O_1$. That is, expert 1 can induce the most preferred action by sending this message. Then, under the self-serving belief, he has an incentive to lie.

(ii) case: By constructing the same deviation in case (i), expert 1 can become strictly better off. That is, the self-serving belief cannot support fully revealing equilibria.

Therefore, if there exists a fully revealing equilibrium supported by the self-serving belief, then this must be case (iii). In other words, it should be $P_1(\theta) \cap P_2(\theta) = \emptyset$.

Proof of Lemma 2

Note that $\overline{L}(\theta, O_2)$ represents the line of endorsement at θ .

(Necessity) Suppose, in contrast, that there exists $\theta \in \Theta$ such that $P_1(\theta) \cap \overline{L}(\theta, O_2) \neq \emptyset$ when the experts have opposing biases. That is, there exists $y \in P_1(\theta) \cap \overline{L}(\theta, O_2)$. As $y \in P_1(\theta)$, $x_1 \cdot \overrightarrow{\theta y} > 0$. In addition, because $y \in \overline{L}(\theta, O_2)$ and the experts have opposing biases, there exists $\alpha \in (0, 1]$ such that $\overrightarrow{\theta y} = \alpha x_2$. However, as the experts have opposing biases, $x_1 \cdot x_2 \leq 0$. That is, $x_1 \cdot (\frac{1}{\alpha} \overrightarrow{\theta y}) \leq 0$, or still $x_1 \cdot \overrightarrow{\theta y} \leq 0$, a contradiction. Therefore, for any state θ , $P_1(\theta) \cap \overline{L}(\theta, O_2) = \emptyset$.

(Sufficiency) Fix $\theta \in \Theta$ and $y \in \overline{L}(\theta, O_2)$ arbitrarily. As $P_1(\theta) \cap \overline{L}(\theta, O_2) = \emptyset$, $y \notin P_1(\theta)$. So, $x_1 \cdot \overrightarrow{\theta y} \leq 0$. In addition, because $y \in \overline{L}(\theta, O_2)$, there exists $\alpha \in [0, 1]$ such that $\overrightarrow{\theta y} = \alpha x_2$. Hence, $x_1 \cdot (\alpha x_2) \leq 0$ implies $x_1 \cdot x_2 \leq 0$. That is, it is an opposing-biases case.

Proof of Proposition 4

(Sufficiency) We show the if the experts have opposing biases, then there exists a fully revealing equilibrium supported by the extended self-serving belief. The proof is constructive; first, we specify expert 2's best response for several s_1 under the extended self-serving belief. Then, we show that expert 1 has no incentive to lie given expert 2's best response. We define the following notation: let $l(s_1)$ and $l(\hat{s}_1)$ be the tangents of $I_1(s_1)$ at $y = s_1$ and \hat{s}_1 , respectively.²¹ Then, given that $s_1 \neq \theta$:

$$s_A \in \arg\min_{y \in I_2(s_1)} ||O_2 - y|| \tag{A.3}$$

$$s_B \in \arg\min_{y \in l(s_1) \cup l(\hat{s}_1)} ||O_2 - y||$$
 (A.4)

$$s_C \in \arg\min_{y \in I_2(s_1) \setminus \mathcal{T}(s_1)} ||O_2 - y|| \tag{A.5}$$

Note that s_A and s_B are uniquely determined as long as $s_1 \neq \theta$. If there are multiple s_C , then we arbitrarily choose one of them.

Similarly to the original case, if $s_1 = \theta$, then expert 2 cannot improve his own payoff by lying. Then suppose that $s_1 \neq \theta$, and divide all cases into the following five cases: (i) $O_2 \notin P_2(s_1) \cup \mathcal{T}(s_1)$, (ii) $O_2 \in P_2(s_1)$ and $s_A \notin P_2(s_1) \cup \mathcal{T}(s_1)$, (iii) $O_2 \in P_2(s_1)$ and $s_A \in P_2(s_1) \cup \mathcal{T}(s_1)$, (iv) $O_2 \notin P_2(s_1)$ and $O_2 \in \mathcal{T}(s_1)$ and $s_B \notin P_2(s_1) \cup \mathcal{T}(s_1)$, and (v) $O_2 \notin P_2(s_1)$ and $O_2 \in \mathcal{T}(s_1)$ and $s_B \notin P_2(s_1) \cup \mathcal{T}(s_1)$, and (v) $O_2 \notin P_2(s_1)$ and $O_2 \in \mathcal{T}(s_1)$ and $s_B \notin P_2(s_1) \cup \mathcal{T}(s_1)$, and (v) $O_2 \notin P_2(s_1) \cup \mathcal{T}(s_1)$.

In Case (i), expert 2's best response is $s_2 = O_2$ and action $y = O_2$ is induced because $O_2 \notin P_1(s_1) \cup \mathcal{T}(s_1)$. In Case (ii), because $O_2 \in P_2(s_1)$, expert 2 cannot induce action $y = O_2$, and

²¹Geometrically, s_B is the foot of the perpendicular from O_2 to either $l(s_1)$ or $l(\hat{s}_1)$.

²²An example of each case is represented in the figures of Appendix B.



Figure A.2: Lemma 3

the most preferable action that expert 2 can induce lies on $I_2(s_1)$. Given $s_1, s_2 = s_A$ is the best response, and action $y = s_A$ is induced because $s_A \notin P_2(s_1) \cup \mathcal{T}(s_1)$. Similarly, as $O_2 \in P_2(s_1)$, the most preferred action for expert 2 that he can induce exists on $I_2(s_1)$ in Case (iii). If he sends $s_2 = s_A$, then action $y = s_1$ is induced because $s_A \in P_2(s_1) \cup \mathcal{T}(s_1)$. By construction, if expert 2 sends $s_2 = s_C$, then it is always believed and either action $y = s_1$ or $y = \hat{s}_1$ is induced. Hence, $s_2 = s_C$ is weakly better than $s_2 = s_A$; it is expert 2's best response.

In Cases (iv) and (v), expert 2's best responses are characterized by the following lemmas.

Lemma 3 Suppose that $O_2 \notin P_2(s_1)$, $O_2 \in \mathcal{T}(s_1)$ and $s_B \notin P_2(s_1) \cup \mathcal{T}(s_1)$. Then, expert 2's best response is $s_2 = s_B$.

Proof of Lemma 3. Because $O_2 \in \mathcal{T}(s_1)$, expert 2 cannot induce action $y = O_2$, given s_1 . Then, expert 2's best response is either $s_2 = s_A$, s_B or s_C .

Case (a): $s_A \in P_2(s_1) \cup \mathcal{T}(s_1)$. It is obvious that if $s_A = s_C$, then $s_A \notin \mathcal{T}(s_1)$. Then, $s_A \neq s_C$ in Case (a). If $s_2 = s_A$, then action $y = s_1$ is induced. As $s_1 \in I_2(s_1) \setminus \mathcal{T}(s_1)$, $s_2 = s_C$ is weakly better for expert 2 than is $s_2 = s_A$. Hence, we compare $||O_2 - s_B||$ with $||O_2 - s_C||$. Without loss of generality, assume that $s_B \in l(s_1)$ and $s_C = s_1$. Because $\overrightarrow{s_BO_2} \cdot \overrightarrow{s_Bs_1} = 0$, $||O_2 - s_B||^2 + ||s_B - s_1||^2 = ||O_2 - s_1||^2$. That is, $||O_2 - s_B|| \leq ||O_2 - s_1||$. Therefore, $s_2 = s_B$ is a best response.

Case (b): $s_A \notin P_2(s_1) \cup \mathcal{T}(s_1)$. As $s_A = s_C$, it is sufficient to compare $||O_2 - s_A||$ and $||O_2 - s_B||$. Without loss of generality, assume that $s_B \in l(s_1)$. Because $s_A \notin \mathcal{T}(s_1)$ and $O_2 \in \mathcal{T}(s_1)$, O_2 and s_A are separated by $l(s_1)$. Then, there exists a point $q \in Y$ such that $\{q\} = l(s_1) \cap \overline{L}(O_2, s_A)$. Let $p \in Y$ be the point such that $p \in l(s_1)$ and $\overrightarrow{ps_A} \cdot \overrightarrow{py} = 0$ for any $y \in l(s_1)$. Note that $\overrightarrow{s_Bq} \cdot \overrightarrow{s_BO_2} = 0$ and $\overrightarrow{pq} \cdot \overrightarrow{ps_A} = 0$. Then, $||O_2 - q||^2 = ||O_2 - s_B||^2 + ||s_B - q||^2$, $||q - s_A||^2 = ||p - s_A||^2 + ||p - q||^2$,



Figure A.3: Lemma 4

and $||O_2 - s_A|| = ||O_2 - q|| + ||q - s_A||$. Then:

$$|O_2 - s_A||^2 > ||O_2 - q||^2 + ||q - s_A||^2$$

= $||O_2 - s_B||^2 + ||s_B - q||^2 + ||p - s_A||^2 + ||p - q||^2$
> $||O_2 - s_B||^2.$ (A.6)

Therefore, $s_2 = s_B$ is one of the best response of expert 2.

Lemma 4 Suppose that $O_2 \notin P_2(s_1), O_2 \in \mathcal{T}(s_1)$ and $s_B \in P_2(s_1) \cup \mathcal{T}(s_1)$. Then, expert 2's best response is $s_2 = s_C$.

Proof of Lemma 4. Let $C(a, L^+(a, b), L^+(a, c))$ be the cone that have a vertex of a and sides of $L^+(a, b)$ and $L^+(a, c)$. Suppose, in contrast, that $O_2 \notin C(O'_2, L^+(O'_2, s_1), L^+(O'_2, \hat{s}_1))$. That is, $O_2 \in \mathcal{T}(s_1) \setminus C(O'_2, L^+(O'_2, s_1), L^+(O'_2, \hat{s}_1))$. There are two possibilities regarding the position of O_2 : either (a) $O_2 \in C(s_1, l(s_1), L^+(O'_2, s_1))$ or (b) $O_2 \in C(\hat{s}_1, l(\hat{s}_1), L^+(O'_2, \hat{s}_1))$. For Case (a), $s_B \in l(s_1) \setminus (\bar{L}(O_{s_1}, s_1) \setminus \{s_1\})$ must be satisfied. This means that $s_B \notin P_2(s_1) \cup \mathcal{T}(s_1)$, a contradiction. For Case (b), we can imply a contradiction in a similar way. Then, $O_2 \in C(O'_2, L^+(O'_2, s_1), L^+(O'_2, \hat{s}_1))$. From Lemma 1, $s_A \in L^+(O'_2, O_2)$. Because $O_2 \in C(O'_2, L^+(O'_2, s_1), L^+(O'_2, \hat{s}_1))$, $s_A \in I_2(s_1) \cap R_1(s_1)$. If $s_A \in I_2(s_1) \cap P_1(s_1)$, then $s_A \neq s_C$ because $P_1(s_1) \subset \mathcal{T}(s_1)$. Then, both $s_2 = s_A$ and $s_2 = s_B$ induce action $y = s_1$. As $s_1 \in I_2(s_1) \setminus \mathcal{T}(s_1)$, $s_2 = s_C$ is best for expert 2. If $s_A \in I_2(s_1) \cap I_1(s_1)$, then $s_A = s_C$. Similarly, as $s_2 = s_B$ induces action $y = s_1$, $s_2 = s_C$ is expert 2's best response. ■

Next, given expert 2's best response specified above, we show that truth telling is a best response for expert 1. If expert 1 sends $s_1 = \theta$, then $s_2 = \theta$ and action $y = \theta$ is induced. Hence, it is sufficient to show that for any deviation, the induced action is not included in $P_1(\theta)$. Consider the same five



Figure A.4: Lemma 5

cases specified above. In Case (i), action $y = O_2$ is induced, but obviously $O_2 \notin P_1(\theta)$ because the experts have opposing biases. The following lemmas show that induced actions are not included in $P_1(\theta)$; Cases (ii), (iii), (iv), and (v) correspond to Lemmas 5, 6, 7, and 8, respectively.

Lemma 5 Suppose that $O_2 \in P_2(s_1)$ and $s_A \notin P_2(s_1) \cup \mathcal{T}(s_1)$. Then, $s_A \notin P_1(\theta)$.

Proof of Lemma 5. By the construction of s_A , $s_A \in I_2(s_1)$. As $s_A \notin \mathcal{T}(s_1)$, $s_A \in I_2(s_1) \setminus \mathcal{T}(s_1)$. That is, $s_A \notin P_1(s_1)$. Suppose, in contrast, that $s_A \in P_1(\theta)$. As $O_2 \in P_2(s_1)$ and by Lemma 1, $s_A \in R_2(\theta)$. That is, $s_A \in R_1(\theta) \cap R_2(\theta)$. Let $e, f \in Y$ be the point such that $\{\theta, e\} = L(\theta, O_2) \cap I_2(\theta)$ where $e \neq \theta$ and $\{\hat{\theta}, f\} = L(\hat{\theta}, O_2) \cap I_2(\theta)$ where $f \neq \hat{\theta}$. From Lemma 1, $s_A \in L^+(O'_2, O_2) \cap I_2(s_1)$. Because of opposing biases, $P_1(\theta) \cap \overline{L}(\theta, O_2) = \emptyset$ and $P_1(\theta) \cap \overline{L}(\hat{\theta}, O_2) = \emptyset$ from Lemma 2. Then, to hold $s_A \in R_1(\theta) \cap R_2(\theta)$, there exist $\alpha, \beta > 0$ such that $\overrightarrow{O_2O'_2} = \alpha \overrightarrow{O_2e} + \beta \overrightarrow{O_2f}$. This implies that there exists point $y^{**} \in Y$ such that $y^{**} \in R_1(\theta) \cap I_2(\theta)$ and $y^{**} + \overrightarrow{O'_2O_2} = s_A$. That is, $s_A \in R_1(s_1) \cap I_2(s_1)$. Because $R_1(s_1) \cap I_2(s_1) \subset R_1(s_1)$, $(R_1(s_1) \cap I_2(s_1)) \setminus \{s_1, \hat{s}_1\} \subset P_1(s_1)$. Because $\alpha, \beta > 0$, $s_A \neq s_1, \hat{s}_1$. Then, $s_A \in (R_1(s_1) \cap I_2(s_1)) \setminus \{s_1, \hat{s}_1\} \subset P_1(s_1)$. That is, we have $s_A \in P_1(s_1)$, which is a contradiction. Therefore, $s_A \notin P_1(\theta)$ must hold.

Lemma 6 Suppose that $O_2 \in P_2(s_1)$ and $s_A \in P_2(s_1) \cup \mathcal{T}(s_1)$. Then, $s_C \notin P_1(\theta)$.

Proof of Lemma 6. As $s_A \in P_2(s_1) \cup \mathcal{T}(s_1)$, $s_A \in P_1(s_1) \cap I_2(s_1)$. Then, there must exist $\alpha, \beta > 0$ such that $\overrightarrow{O'_2O_2} = \alpha \overrightarrow{O'_2s_1} + \beta \overrightarrow{O'_2s_1}$. As $\overrightarrow{O_\theta O_{s_1}} = \overrightarrow{O_2O'_2}$, $\overrightarrow{O_\theta O_{s_1}} = -\alpha \overrightarrow{O'_2s_1} - \beta \overrightarrow{O'_2s_1}$. Hence:

$$\overrightarrow{O_{\theta}O_{s_1}} = -\alpha \overrightarrow{O_2's_1} - \beta \overrightarrow{O_2'\hat{s_1}} = -\alpha (\overrightarrow{O_2'O_{s_1}} + \overrightarrow{O_{s_1}\hat{s_1}}) - \beta (\overrightarrow{O_2'O_{s_1}} + \overrightarrow{O_{s_1}\hat{s_1}})$$
$$= -\alpha \overrightarrow{O_{s_1}\hat{s_1}} - \beta \overrightarrow{O_{s_1}\hat{s_1}} - (\alpha + \beta) \overrightarrow{O_2'O_{s_1}}.$$
(A.7)

As the experts have opposing biases, there exist $\gamma, \delta \geq 0$ such that $\overrightarrow{O_{s_1}O'_2} = -\gamma \overrightarrow{O_{s_1}s_1} - \delta \overrightarrow{O_{s_1}s_1}$. Then:

$$\overrightarrow{O_{\theta}O_{s_1}} = (-\alpha - \gamma(\alpha + \beta))\overrightarrow{O_{s_1}s_1} + (-\beta - \delta(\alpha + \beta))\overrightarrow{O_{s_1}\hat{s_1}}$$

$$= (-\alpha - \gamma(\alpha + \beta))\overrightarrow{O_{\theta}\theta} + (-\beta - \delta(\alpha + \beta))\overrightarrow{O_{\theta}\hat{\theta_1}}.$$
(A.8)

Claim 1 Suppose that there exists $\alpha, \beta < 0$ such that $\overrightarrow{O_{\theta}O_{s_1}} = \alpha \overrightarrow{O_{\theta}\theta} + \beta \overrightarrow{O_{\theta}\theta}$. Then, $C(O_{\theta}, L^+(O_{\theta}, \theta), L^+(O_{\theta}, \hat{\theta})) \subseteq \mathcal{T}(s_1)$.

Proof of Claim 1. Suppose that $\alpha, \beta < 0$. Take any $y \in C(O_{\theta}, L^{+}(O_{\theta}, \theta), L^{+}(O_{\theta}, \hat{\theta}))$. Then, there exist $\gamma_{1}, \delta_{1} \geq 0$ such that $\overrightarrow{O_{\theta}y} = \gamma_{1}\overrightarrow{O_{\theta}\theta} + \delta_{1}\overrightarrow{O_{\theta}\theta}$. Hence:

$$\overrightarrow{O_{s_1}y} = \overrightarrow{O_{s_1}O_{\theta}} + \overrightarrow{O_{\theta}y} = -\alpha \overrightarrow{O_{\theta}\theta} - \beta \overrightarrow{O_{\theta}\theta} + \gamma_1 \overrightarrow{O_{\theta}\theta} + \delta_1 \overrightarrow{O_{\theta}\theta}$$

$$= (-\alpha + \gamma_1)\overrightarrow{O_{s_1}s_1} + (-\beta + \delta_1)\overrightarrow{O_{s_1}s_1}.$$
(A.9)

As $-\alpha + \gamma_1, -\beta + \delta_1 > 0, y \in \mathcal{T}(s_1)$. Therefore, $C(O_{\theta}, L^+(O_{\theta}, \theta), L^+(O_{\theta}, \hat{\theta})) \subseteq \mathcal{T}(s_1)$. \Box As $-\alpha - \gamma(\alpha + \beta), -\beta - \delta(\alpha + \beta) < 0, C(O_{\theta}, L^+(O_{\theta}, \theta), L^+(O_{\theta}, \hat{\theta})) \subseteq \mathcal{T}(s_1)$ from Claim 1. Since $s_C \notin \mathcal{T}(s_1), s_C \notin C(O_{\theta}, L^+(O_{\theta}, \theta), L^+(O_{\theta}, \hat{\theta}))$. Because $P_1(\theta) \subset C(O_{\theta}, L^+(O_{\theta}, \theta), L^+(O_{\theta}, \hat{\theta})),$ $s_C \notin P_1(\theta)$.

Lemma 7 Suppose that $O_2 \notin P_2(s_1)$, $O_2 \in \mathcal{T}(s_1)$ and $s_B \notin P_2(s_1) \cup \mathcal{T}(s_1)$. Then, $s_B \notin P_1(\theta)$.

Proof of Lemma 7. Let $\mathcal{T}(\theta)$ be the interior of the cone $C(O_{\theta}, L^+(O_{\theta}, \theta), L^+(O_{\theta}, \hat{\theta}))$. As the experts have opposing biases, $O_2 \notin \mathcal{T}(\theta)$. Because $O_2 \notin \mathcal{T}(\theta)$ and $O_2 \in \mathcal{T}(s_1)$, there exist $\alpha, \beta < 0$ such that $\overrightarrow{O_{\theta}O_{s_1}} = \alpha \overrightarrow{O_{\theta}\theta} + \beta \overrightarrow{O_{\theta}\theta}$. From Claim 1, $C(O_{\theta}, L^+(O_{\theta}, \theta), L^+(O_{\theta}, \hat{\theta})) \subseteq \mathcal{T}(s_1)$. As $s_B \notin \mathcal{T}(s_1)$, $s_B \notin C(O_{\theta}, L^+(O_{\theta}, \theta), L^+(O_{\theta}, \hat{\theta}))$. Because $P_1(\theta) \subset C(O_{\theta}, L^+(O_{\theta}, \theta), L^+(O_{\theta}, \hat{\theta}))$, $s_B \notin P_1(\theta)$.

Lemma 8 Suppose that $O_2 \notin P_2(s_1)$, $O_2 \in \mathcal{T}(s_1)$ and $s_B \in P_2(s_1) \cup \mathcal{T}(s_1)$. Then, $s_C \notin P_1(\theta)$.

Proof of Lemma 8. As we have shown in the proof of Lemma 4, $O_2 \in C(O'_2, L^+(O'_2, s_1), L^+(O'_2, \hat{s}_1))$. Also, from the proof of Lemma 4, we can say that $s_A \in R_1(s_1) \cap I_2(s_1)$. Suppose, in contrast, that $s_A \in I_1(s_1) \cap I_2(s_1)$; that is, $s_A = s_C \in \{s_1, \hat{s}_1\}$. This means that either $O_2 \in L(O'_2, s_1)$ or $O_2 \in L(O'_2, \hat{s}_1)$. However, as we have shown in the proof of Lemma 4, this implies that $s_B \notin P_2(s_1) \cup \mathcal{T}(s_1)$ because $O_2 \in \mathcal{T}(s_1) \cap C(O'_2, L^+(O'_2, s_1), L^+(O'_2, \hat{s}_1))$, which is a contradiction. Hence, $s_A \in P_1(s_1) \cap I_2(s_1)$ must hold. We can apply the same argument in the proof of Lemma 6, and then we can say that $s_C \notin P_1(\theta)$.

By Lemmas 5, 6, 7, and 8, we can say that for any state θ , any deviation from $s_1 = \theta$ never improves expert 1's payoff. Thus, given expert 2's and the decision-maker's strategies, truth telling is one of the best response of expert 1. It is obvious that, on the equilibrium path, the belief specified by the extended self-serving belief is consistent with Bayes' rule. Therefore, it is a PBE, a fully revealing equilibrium. In summary, the players' strategies are described as follows:

$$\mu_{1}^{*}(\theta) = \theta \qquad (A.10)$$

$$\mu_{2}^{*}(\theta, s_{1}) = \begin{cases} \theta & \text{if } s_{1} = \theta \\ O_{2} & \text{if } O_{2} \notin P_{2}(s_{1}) \cup \mathcal{T}(s_{1}) \\ s_{A} & \text{if } O_{2} \in P_{2}(s_{1}) \text{ and } s_{A} \notin P_{2}(s_{1}) \cup \mathcal{T}(s_{1}) \\ s_{C} & \text{if } O_{2} \in P_{2}(s_{1}) \text{ and } s_{A} \in P_{2}(s_{1}) \cup \mathcal{T}(s_{1}) \\ s_{B} & \text{if } O_{2} \notin P_{2}(s_{1}) \text{ and } O_{2} \in \mathcal{T}(s_{1}) \text{ and } s_{B} \notin P_{2}(s_{1}) \cup \mathcal{T}(s_{1}) \\ s_{C} & \text{if } O_{2} \notin P_{2}(s_{1}) \text{ and } O_{2} \in \mathcal{T}(s_{1}) \text{ and } s_{B} \notin P_{2}(s_{1}) \cup \mathcal{T}(s_{1}) \\ s_{C} & \text{if } O_{2} \notin P_{2}(s_{1}) \cup \mathcal{T}(s_{1}) \text{ and } \theta = s_{2} \\ 0 & \text{if } s_{2} \notin P_{2}(s_{1}) \cup \mathcal{T}(s_{1}) \text{ and } \theta \neq s_{2} \\ 1 & \text{if } s_{2} \in P_{2}(s_{1}) \cup \mathcal{T}(s_{1}) \text{ and } \theta \neq s_{2} \\ 1 & \text{if } s_{2} \in P_{2}(s_{1}) \cup \mathcal{T}(s_{1}) \text{ and } \theta \neq s_{1} \\ 0 & \text{if } s_{2} \in P_{2}(s_{1}) \cup \mathcal{T}(s_{1}) \text{ and } \theta \neq s_{1} \\ y^{*}(s_{1}, s_{2}) = \begin{cases} s_{2} & \text{if } s_{2} \notin P_{2}(s_{1}) \cup \mathcal{T}(s_{1}) \\ s_{1} & \text{if } s_{2} \in P_{2}(s_{1}) \cup \mathcal{T}(s_{1}) \end{cases} \qquad (A.13)$$

(Necessity) Suppose, by contrast, that there exists a fully revealing equilibrium $(\mu_1^*, \mu_2^*, y^*; \mathcal{P}^*)$ in like-biases cases. By Lemma 2, $P_1(\theta) \cap \overline{L}(\theta, O_2) \neq \emptyset$. Pick a point $\theta' \in P_1(\theta) \cap \overline{L}(\theta, O_2)$, as in Figure 11. Because there exists a fully revealing equilibrium, on the equilibrium path, there exist messages s_1, s_2, s'_1 and s'_2 such that:

 \mathcal{P}^{i}

$$\mu_1^*(\theta) = s_1, \ \mu_2^*(\theta, s_1) = s_2, \ y^*(s_1, s_2) = \theta,$$
(A.14)

$$\mu_1^*(\theta') = s_1', \ \mu_2^*(\theta', s_1') = s_2', \ y^*(s_1', s_2') = \theta'.$$
(A.15)

First, show that $s_1 \neq s'_1$. Suppose, in contrast, that $s_1 = s'_1$. Because $y^*(s_1, s_2) \neq y^*(s'_1, s'_2)$, $s_2 \neq s'_2$. However, because $\theta' \in \overline{L}(\theta, O_2)$, expert 2 has an incentive to send s'_2 at state θ after observing s_1 , which is a contradiction. Therefore, $s_1 \neq s'_1$ must hold. Next, we show that, given expert 1's message s'_1 , there is no message s''_2 by expert 2 such that $y^*(s'_1, s''_2) \in P_2(\theta')$. Suppose, by contrast, that there exists such a message s''_2 . Then, expert 2's best response to the message s'_1 at θ' is not sending s'_2 because, by sending s''_2 , expert 2's utility is strictly improved. This contradicts the message s'_2 being on the equilibrium path. Thus, such a message s''_2 does not exist. Finally, we show that expert 1 has an incentive to deviate. Suppose that expert 1 sends message s'_1 at the state θ . Given s'_1 , there is no message s''_2 such that $y^*(s'_1, s''_2) \in P_2(\theta')$, as shown above. That is, expert 2 cannot induce the actions in $P_2(\theta')$ if expert 1 sends message s'_1 . From the construction of θ' and Lemma 1, the most preferred action in $\mathbb{R}^2 \setminus P_2(\theta')$ for expert 2 is $y = \theta'$, and it can be induced by sending s'_2 . That is, sending message s'_2 is expert 2's unique best response to message s'_1 at state θ . However, action $y = \theta' \in P_1(\theta)$. This means that expert 1 has no incentive to send the message s_1 , which thus contradicts the message s_1 being on the equilibrium path. Therefore, there is no fully revealing equilibrium.²³

Proof of Proposition 5

Suppose, in contrast, that there exists a fully revealing equilibrium even if the conditions (i) and (ii) hold. Fix $\theta \in \Theta$ arbitrarily, and choose $\theta' \in \Theta$ satisfying the conditions. Because there exists a fully revealing equilibrium, there exist the following messages:

$$\mu_1^*(\theta) = s_1, \ \mu_2^*(\theta, s_1) = s_2, \ \mu_3^*(\theta, s_1, s_2) = s_3, \ y^*(s_1, s_2, s_3) = \theta;$$
(A.16)

$$\mu_1^*(\theta') = s_1', \ \mu_2^*(\theta', s_1') = s_2', \ \mu_3^*(\theta', s_1', s_2') = s_3', \ y^*(s_1', s_2', s_3') = \theta'.$$
(A.17)

Note that given s'_1 and s'_2 , for any $\tilde{s}_3 \in S_3$, $y^*(s'_1, s'_2, \tilde{s}_3) \notin P_3(\theta')$; otherwise, expert 3 deviates from sending s'_3 at state θ' . By Lemma 1 and conditions (i), sending s'_3 is expert 3's unique best response after observing s'_1 and s'_2 at state θ .

Claim 2 $s_1 \neq s'_1$.

Proof of Claim 2. Suppose, in contrast, that $s_1 = s'_1 = s^*$. Because $y^*(s_1, s_2, s_3) \neq y^*(s'_1, s'_2, s'_3)$, $(s_2, s_3) \neq (s'_2, s'_3)$. If $s_2 = s'_2$, then $s_3 \neq s'_3$ must hold. However, in this scenario, expert 3 has an incentive to send s'_3 instead of s_3 at state θ because θ' lies in his line of endorsement at θ . That is, $s_2 \neq s'_2$ must hold. By Condition (ii), because $\theta \notin P_1(\theta), \theta \notin K_2(\theta, \theta')$. Now, given that expert 1 sends s^* at state θ , consider a situation where expert 2 sends s'_2 . Because expert 3 sends s'_3 after

²³Notice that the proof of the necessary part does not depend on the direct message game setting. That is, the impossibility result holds in both direct and indirect message games.

observing s^* and s'_2 at state θ , action $y = \theta'$ is induced. Because $\theta' \in K_2(\theta, \theta')$ and $\theta \notin K_2(\theta, \theta')$, this is a profitable deviation for expert 2, which is a contradiction. Therefore, $s_1 \neq s'_1$. \Box

By Claim 2, sending s'_1 at state θ is a deviation by expert 1. Consider expert 2's response given observing s'_1 at state θ . Suppose that $y^*(s'_1, \tilde{s}_2, \mu_3(\theta, s'_1, \tilde{s}_2)) \notin K_2(\theta, \theta')$ for any $\tilde{s}_2 \in S_2 \setminus \{s'_2\}$. Because expert 3 sends s'_3 after observing s'_1 and s'_2 at state θ and $\theta' \in K_2(\theta, \theta')$, sending s'_2 is expert 2's unique best response in this scenario. Hence, action $y = \theta'$ is induced. However, because $\theta' \in K_2(\theta, \theta')$, by Condition (ii), sending s'_1 at state θ is expert 1's profitable deviation. Therefore, to hold the fully revealing equilibrium, there must exists message $\hat{s}_2 \in S_2 \setminus \{s'_2\}$ such that $\mu_2^*(\theta, s'_1) = \hat{s}_2$ and $y^*(s'_1, \hat{s}_2, \mu_3(\theta, s'_1, \hat{s}_2)) \in K_2(\theta, \theta')$. Also, to hold the fully revealing equilibrium, $y^*(s'_1, \hat{s}_2, \mu_3(\theta, s'_1, \hat{s}_2)) \notin P_1(\theta)$; otherwise, expert 1 has an incentive to deviate. However, by Condition (ii), $y^*(s'_1, \hat{s}_2, \mu_3(\theta, s'_1, \hat{s}_2)) \in K_2(\theta, \theta') \setminus P_1(\theta)$ is impossible, which is a contradiction. Therefore, there exists no fully revealing equilibrium.

Proof of Proposition 6

We modify the strategies and beliefs of the experts as follows. Let $\mu_1 : \Sigma \to S_1$ represent expert 1's pure strategy, and $\mu_2 : \Sigma \times S_1 \to S_2$ and $\mathcal{P}_2 : \Sigma \times S_1 \to \Delta(\Theta)$ represent expert 2's pure strategy and belief, respectively.²⁴ Define set $J \equiv \{y \in Y | \text{ there exist } \alpha_1 \in \mathbb{R} \text{ and } \alpha_2 \leq 0 \text{ such that } y = O_2 + \alpha_1 \overrightarrow{O_{\theta} \theta} + \alpha_2 \overrightarrow{O_{\theta} \theta} \}$. That is, set J is the half-space separated by line \overline{l} , which is parallel to $l(\theta)$ and go through O_2 . Define $s_D \equiv L(O_1, \theta) \cap \overline{l}$.

(i) Suppose that $x_1 \cdot x_2 < 0$, $\epsilon_1 = 0$ and $\epsilon_2 < 1 - \frac{|U^{E_1}(\theta, \theta, x_1)|}{|U^{E_1}(s_D, \theta, x_1)|}$.²⁵ Let $(\mu_1^*, \mu_2^*, y^*; \mathcal{P}^*)$ be the fully revealing equilibrium specified in Proposition 4, and show that $(\tilde{\mu}_1, \tilde{\mu}_2, \tilde{y}; \tilde{\mathcal{P}}_2, \tilde{\mathcal{P}})$ defined as follows is a PBE.

$$\tilde{\mu}_1(\theta) = \theta \tag{A.18}$$

$$\tilde{\mu}_2(\sigma, s_1) = \begin{cases} \mu_2^*(\theta, s_1) & \text{if } \sigma = \theta \\ s_1 & \text{if } \sigma = \phi \end{cases}$$
(A.19)

$$\tilde{y}(s_1, s_2) = y^*(s_1, s_2)$$
 (A.20)

$$\tilde{\mathcal{P}}_{2}(\tilde{\theta}|\sigma, s_{1}) = \begin{cases} 1 & \text{if } [\sigma = \theta \text{ and } \tilde{\theta} = \theta] \text{ or } [\sigma = \phi \text{ and } \tilde{\theta} = s_{1}] \\ 0 & \text{otherwise} \end{cases}$$
(A.21)

$$\tilde{\mathcal{P}}(\tilde{\theta}|s_1, s_2) = \mathcal{P}^*(\tilde{\theta}|s_1, s_2) \tag{A.22}$$

²⁵Note that because the experts have opposing biases, $0 < \frac{|U^{E_1}(\theta, \theta, x_1)|}{|U^{E_1}(s_D, \theta, x_1)|} < 1.$

²⁴Expert 1's belief is uniquely determined upon his observation. If $\sigma_1 = \theta$, then the belief assigns probability 1 to the true state. If $\sigma_1 = \phi$, then the belief is identical to the prior distribution. For simple exposition, we omit the representation of expert 1's belief.



Figure A.5: Proposition 6

It is straightforward that \tilde{y} is the decision-maker's best response given her belief \tilde{P} . We consider expert 2's decision given his belief $\tilde{\mathcal{P}}_2$ and \tilde{y} . If $\sigma = \theta$, then this scenario is identical to the noiseless case. Hence, by the same arguments in Proposition 4, μ_2^* represents the optimal behavior of expert 2. If $\sigma = \phi$, then expert 2 believes that expert 1 reports the true state for certain. Given this belief and \tilde{y} consistent with the extended self-serving belief, sending $s_2 = s_1$ is optimal because expert 2 cannot strictly improve his utility by sending $s_2 \neq s_1$. Hence, $\tilde{\mu}_2$ represents expert 2's best response.

Next, we consider expert 1's decision given $\tilde{\mu}_2$ and \tilde{y} . If expert 1 sends $s_1 = \theta$, then action $y = \theta$ is induced. That is, his utility is $U^{E_1}(\theta, \theta, x_1)$. If expert 1 sends $s_1 \neq \theta$, then his expected utility is $\tilde{U}(s_1) \equiv (1 - \epsilon_2)U^{E_1}(y^*(s_1, \mu_2^*(\theta, s_1)), \theta, x_1) + \epsilon_2U^{E_1}(s_1, \theta, x_1)$. Now, we suppose that $s_1 \notin P_1(\theta)$. Then, $U^{E_1}(\theta, \theta, x_1) \geq \tilde{U}(s_1)$ because $y^*(s_1, \mu_2^*(\theta, s_1)) \notin P_1(\theta)$ as shown in Proposition 4. Therefore, expert 1 has no strict incentive to send $s_1 \notin P_1(\theta)$. Then, we suppose that $s_1 \in P_1(\theta)$. The next lemma gives us an upper bound of $\tilde{U}(s_1)$.

Lemma 9 Suppose that $x_1 \cdot x_2 < 0$. Then, $U^{E_1}(s_D, \theta, x_1) \ge U^{E_1}(y^*(s_1, \mu_2(\theta, s_1)), \theta, x_1)$ holds for any $\theta \in \Theta$ and $s_1 \in P_1(\theta)$.

Proof of Lemma 9. Fix $\theta \in \Theta$ and $s_1 \in P_1(\theta)$ arbitrarily. First, we show that $y^*(s_1, \mu_2^*(\theta, s_1)) = s_A$ or O_2 . It is obvious that $y^*(s_1, \mu_2^*(\theta, s_1)) = O_2$ if $O_2 \notin P_2(s_1) \cup \mathcal{T}(s_1)$. Then, we assume that $O_2 \in P_2(s_1) \cup \mathcal{T}(s_1)$. Because $x_1 \cdot x_2 < 0$, there exist $\beta_1, \beta_2 < 0$ such that $\overrightarrow{O_\theta O_2} = \beta_1 \overrightarrow{O_\theta \theta} + \beta_2 \overrightarrow{O_\theta \theta}$. Also, because $s_1 \in P_1(\theta)$, there exist $\gamma_1, \gamma_2 > 0$ such that $\overrightarrow{\theta s_1} = \gamma_1 \overrightarrow{O_\theta \theta} + \gamma_2 \overrightarrow{O_\theta \theta}$. Therefore, if $O_2 \in \mathcal{T}(s_1)$, then $\gamma_2 < 0$ must hold, but it is impossible as long as $s_1 \in P_1(\theta)$. Thus, $O_2 \in P_2(s_1) \setminus \mathcal{T}(s_1)$. Now, we suppose, in contrast, that $s_A \in \mathcal{T}(s_1)$. By Lemma 1, if $s_A \in \mathcal{T}(s_1)$, then there exist $\delta_1, \delta_2 \leq 0$ such that $\overrightarrow{\theta s_1} = \delta_1 \overrightarrow{O_2 \theta} + \delta_2 \overrightarrow{O_2 \theta}$. Hence:

$$\overrightarrow{\theta s_1} = \delta_1(\overrightarrow{O_2O_\theta} + \overrightarrow{O_\theta\theta}) + \delta_2(\overrightarrow{O_2O_\theta} + \overrightarrow{O_\theta\theta})$$
$$= (-\beta_1\delta_1 - \beta_1\delta_2 + \delta_1)\overrightarrow{O_\theta\theta} + (-\beta_2\delta_1 - \beta_2\delta_2 + \delta_2)\overrightarrow{O_\theta\theta}.$$
(A.23)

However, because $-\beta_2\delta_1 - \beta_2\delta_2 + \delta_2 \leq 0$, which is a contradiction to $\gamma_2 > 0$. Then, $s_A \notin \mathcal{T}(s_1)$. Therefore, if $O_2 \in P_2(s_1) \cup \mathcal{T}(s_1)$, then $y^*(s_1, \mu_2(\theta, s_1)) = s_A$.

Next, we show that $y^*(s_1, \mu_2(\theta, s_1)) \in J$. By the above arguments, if $s_1 \in P_1(\theta)$, then $y^*(s_1, \mu_2(\theta, s_1)) = s_A$ or O_2 . Because it is obvious that $O_2 \in J$, it is sufficient to show that $s_A \in J$. By Lemma 1, $s_A \in I_2(s_1) \cap L^+(O'_2, O_2)$; that is, there exists $\eta > 0$ such that $s_A = O_2 - \eta \overrightarrow{\theta s_1}$. Hence, $s_A = O_2 - \eta \gamma_1 \overrightarrow{O_\theta \theta} - \eta \gamma_2 \overrightarrow{O_\theta \theta}$. Because $-\eta \gamma_2 < 0$, $s_A \in J$. By construction, $||O_1 - J|| = ||O_1 - s_D||$. Because $s_D \in \overline{l}$ and \overline{l} goes through O_2 , we can say that $U^{E_1}(s_D, \theta, x_1) \ge U^{E_1}(y^*(s_1, \mu_2^*(\theta, s_1)), \theta, x_1)$.

By Lemma 9:

$$U^{E_{1}}(\theta,\theta,x_{1}) - (1-\epsilon_{2})U^{E_{1}}(y^{*}(s_{1},\mu_{2}(\theta,s_{1})),\theta,x_{1}) - \epsilon_{2}U^{E_{1}}(s_{1},\theta,x_{1})$$

$$= (1-\epsilon_{2})\{U^{E_{1}}(\theta,\theta,x_{1}) - U^{E_{1}}(y^{*}(s_{1},\mu_{2}(\theta,s_{1})),\theta,x_{1})\} + \epsilon_{2}\{U^{E_{1}}(\theta,\theta,x_{1}) - U^{E_{1}}(s_{1},\theta,x_{1})\}$$

$$\geq (1-\epsilon_{2})\{U^{E_{1}}(\theta,\theta,x_{1}) - U^{E_{1}}(s_{D},\theta,x_{1})\} + \epsilon_{2}\{U^{E_{1}}(\theta,\theta,x_{1}) - U^{E_{1}}(O_{1},\theta,x_{1})\}$$

$$= (1-\epsilon_{2})\{U^{E_{1}}(\theta,\theta,x_{1}) - U^{E_{1}}(s_{D},\theta,x_{1})\} + \epsilon_{2}U^{E_{1}}(\theta,\theta,x_{1}).$$
(A.24)

Because $\epsilon_2 < 1 - \frac{|U^{E_1}(\theta, \theta, x_1)|}{|U^{E_2}(s_D, \theta, x_1)|}$ and $x_1 \cdot x_2 < 0$, equation (A.24) is positive. Therefore, expert 1 has no strict incentive to send $s_1 \in P_1(\theta)$. Thus, $\tilde{\mu}_1$ is expert 1's best response. It is straightforward that $\tilde{\mathcal{P}}_2$ and $\tilde{\mathcal{P}}$ are consistent with Bayes' rule. Therefore, this is a PBE, and the decision-maker always knows the true state on the equilibrium path. That is, it is a fully revealing equilibrium.

(b) Suppose, in contrast, that there exists a fully revealing equilibrium $(\mu_1^*, \mu_2^*, y^*; \mathcal{P}_2^*, \mathcal{P})$ for some $\epsilon_1 > 0$. That is, for any $\theta \in \Theta$, $(\sigma_1, \sigma_2) = (\phi, \theta)$ occurs with positive probability, and then $y^*(\mu_1^*(\phi), \mu_2(\theta, \mu_1^*(\phi))) = \theta$ must hold. Fix $\theta \in \Theta$, arbitrarily, and define $\theta' \equiv \theta + x_2$ and $\hat{s}_2 \equiv$ $\mu_2^*(\theta', \mu_1^*(\phi))$. By definition of fully revealing equilibrium, $y^*(\mu_1^*(\phi), \hat{s}_2) = \theta'$ and $y^*(\mu_1^*(\phi), \mu_2^*(\theta, \mu_2^*(\phi))) =$ θ must hold. Hence, it must be $\mu_2^*(\theta, \mu_1^*(\phi)) \neq \hat{s}_2$. However, given y^* , expert 2 who observes $\sigma_2 = \theta$ and $s_1 = \mu_1^*(\phi)$ has an incentive to deviate from $s_2 = \mu_2^*(\theta, \mu_1^*(\phi))$ to $s_2 = \hat{s}_2$ because the latter induces the most favorite action to expert 2, which is a contradiction. Therefore, there exists no fully revealing equilibrium for any $\epsilon_1 \in (0, 1)$ and $\epsilon_2 \in [0, 1)$.



Appendix B: Figures of Proposition 4. Cases 1 to 6.

Appendix C: Supplementary Materials

C.1 *n*-expert models

Claim 3 Consider the three-expert model. If the conditions of Proposition 5 are satisfied, then they have like biases.

Proof. Suppose, in contrast, that they have opposing biases, and arbitrarily fix θ . By Condition (ii), because $K_2(\theta, \theta') \subset P_1(\theta)$, $O_2 \in P_1(\theta)$. That is, $x_1 \cdot x_2 > 0$. Thus, one of the following cases must occur: (i) $x_1 \cdot x_3 \leq 0$, or (ii) $x_1 \cdot x_3 > 0$ and $x_2 \cdot x_3 \leq 0$.

In the first scenario, because $x_1 \cdot x_3 \leq 0$, $\tilde{\theta} \notin P_1(\theta)$ for any $\tilde{\theta} \in \bar{L}(\theta, O_3)$. By Condition (i), $\theta' \in \bar{L}(\theta, O_3)$, and then $\theta' \notin P_1(\theta)$. However, by definition, $\theta' \in K_2(\theta, \theta')$ must hold, which is a contradiction to Condition (ii). In the second scenario, because $x_2 \cdot x_3 \leq 0$, $P_2(\theta) \subseteq K_2(\theta, \tilde{\theta})$ for any $\tilde{\theta} \in \bar{L}(\theta, O_3)$. By Conditions (i) and (ii), we can say that $P_2(\theta) \subseteq K_2(\theta, \theta') \subset P_1(\theta)$. To hold this relation, x_1 and x_2 must be linearly dependent. However, because $x_1 \cdot x_3 > 0$, $x_2 \cdot x_3 > 0$ must hold, which is a contradiction. Therefore, this case must be like biases.

C.2 Mixed strategies

With abuse of notation, let $\mu_1 : \Theta \to \Delta(S_1)$ and $\mu_2 : \Theta \times S_1 \to \Delta(S_2)$ be experts 1's and 2's strategies, respectively. Note that the decision-maker always undertakes a pure strategy because of the quadratic-loss utility function. We say that a PBE is a *fully revealing equilibrium* in the mixed-strategy environment if for any $\theta \in \Theta$, any $s_1 \in supp(\mu_1^*(\theta))$ and any $s_2 \in supp(\mu_2^*(\theta, s_1))$, $\mathcal{P}^*(\theta|s_1, s_2) = 1$.

Proposition 8 Consider the two-dimensional unbounded state space model with mixed strategies. Then, there exists a fully revealing equilibrium if and only if the experts have opposing biases.

Proof. (Sufficiency) Show that the strategies and belief specified in Proposition 4 is also an equilibrium in the mixed-strategy environment. It is straightforward that y^* is decision-maker's best response given \mathcal{P}^* . Because expert 2 observes the exact message that expert 1 sends before sending a message, μ_2^* specified is expert 2's best response. Because sending $s_1 \neq \theta$ does not improve expert 1's utility given μ_2^* and y^* as shown in Lemmas 5 to 8, expert 1 has no incentive to randomize messages other than $s_1 = \theta$. Therefore, μ_1^* is expert 1's best response. It is obvious that, on the equilibrium path, the belief specified by \mathcal{P}^* is consistent with Bayes' rule. Therefore, it is a fully revealing equilibrium. (Necessity) Suppose, in contrast, that there exists a fully revealing equilibrium when the experts have like biases. Fix $\theta \in \Theta$ arbitrarily. By Lemma 2, $P_1(\theta) \cap \overline{L}(\theta, O_2) \neq \emptyset$. Choose $\theta' \in P_1(\theta) \cap \overline{L}(\theta, O_2)$ arbitrarily. Because there exists a fully revealing equilibrium $(\mu_1^*, \mu_2^*, y^*; \mathcal{P}^*)$, there exist a pair of messages (s_1, s_2) and (s'_1, s'_2) such that:

 $s_1 \in supp(\mu_1^*(\theta)), \ s_2 \in supp(\mu_2^*(\theta, s_1)), \ y^*(s_1, s_2) = \theta,$ (A.25)

$$s_1' \in supp(\mu_1^*(\theta')), \ s_2' \in supp(\mu_2^*(\theta', s_1')), \ y^*(s_1', s_2') = \theta',$$
(A.26)

Claim 4 $s'_1 \notin supp(\mu_1^*(\theta)).$

Proof of Claim 4. Suppose, in contrast, that $s'_1 \in supp(\mu_1^*(\theta))$. Because $(\mu_1^*, \mu_2^*, y^*; \mathcal{P}^*)$ is a fully revealing equilibrium, there exists a message $\hat{s}_2 \in S_2$ such that $\hat{s}_2 \in supp(\mu_2^*(\theta, s'_1))$ and $y^*(s'_1, \hat{s}_2) = \theta$. Hence, $U^{E_2}(\theta, \theta, x_2) \geq U^{E_2}(y^*(s'_1, \tilde{s}_2), \theta, x_2)$ for all $\tilde{s}_2 \in S_2$ must hold; otherwise, it contradicts that $\hat{s}_2 \in supp(\mu_2^*(\theta, s'_1))$. However, because $\theta' \in \bar{L}(\theta, O_2)$ and $y^*(s'_1, s'_2) = \theta'$, $U^{E_2}(y^*(s'_1, s'_2), \theta, x_2) > U^{E_2}(\theta, \theta, x_2)$, which is a contradiction. \Box

Claim 5 $\mu_2^*(\theta, s_1') = s_2'$.

Proof of Claim 5. Note that for any $\tilde{s}_2 \in S_2$, $y^*(s'_1, \tilde{s}_2) \notin P_2(\theta')$; otherwise, expert 2 never sends s'_2 at state θ' . Because $\theta' \in \bar{L}(\theta, O_2)$ and by Lemma 1, sending s'_2 is expert 2's unique best response when observing s'_1 at state θ . That is, $\mu_2^*(\theta, s'_1) = s'_2$. \Box

By Claim 4, sending s'_1 at state θ is a deviation from μ_1^* . By Claim 5, expert 2 sends s'_2 after observing s'_1 at state θ . That is, action $y = \theta'$ is induced. However, because $\theta' \in P_1(\theta)$, this is a profitable deviation for expert 1, which is a contradiction. Therefore, there exists no fully revealing equilibrium when the experts have like biases.

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